

Non-Abelian Gauge Family Symmetry in Rank 8 and 16 Grand Unified String Theories

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ABSTRACT

The one of the main points of the investigations in high energy physics is to study the next chain: a law of the quark and lepton mass spectra \rightarrow the puzzles of the quark and lepton family mixing \rightarrow a possible new family dynamics.

The new family symmetry dynamics might be connected to the existence of some exotic gauge or matter fields or something yet. For this it will better to study the possibilities of the appearance this gauge symmetry in the framework of the Grand Unified String Theories. In the framework of four dimensional heterotic superstring with free fermions we investigate the rank eight Grand Unified String Theories (GUST) which contain the $SU(3)_H$ -gauge family symmetry. We explicitly construct GUST with gauge symmetry $G = SU(5) \times U(1) \times (SU(3) \times U(1))_H$ and $G = SO(10) \times (SU(3) \times U(1))_H \subset SO(16)$ or $E(6) \times SU(3)_H \subset E(8)$ in free complex fermion formulation. As the GUSTs originating from Kac-Moody algebras (KMA) contain only low-dimensional representations it is usually difficult to break the gauge symmetry. We solve this problem taking for the observable gauge symmetry the diagonal subgroup G^{sym} of rank 16 group $G \times G \subset SO(16) \times SO(16)$ or $(E(6) \times SU(3)_H)^2 \subset E(8) \times E(8)$. We discuss the possible fermion matter and Higgs sectors in these models. In these GUST there has to exist "superweak" light chiral matter ($m_H^f < M_W$). The understanding of quark and lepton mass spectra and family mixing leave a possibility for the existence of an unusually low mass breaking scale of the $SU(3)_H$ family gauge symmetry (some TeV).

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1 Theoretical trends beyond the SM

1.1 The family mixing state in Standard Model

There are no experimental indications which would impel one to go beyond the framework of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ Standard Model (SM) with three generations of quarks and leptons. None of the up-to-date experiments contradict, within the limits of accuracy, the validity of the SM predictions for low energy phenomena. The fermion mass origin and generation mixing, CP-violation problems are among most exciting theoretical puzzles in SM.

One has ten parameters in the quark sector of the SM with three generations: six quark masses, three mixing angles and the Kobayashi-Maskawa (KM) CP- violation phase ($0 < \delta^{KM} < \pi$). The CKM matrix in Wolfenstein parametrization is determined by the four parameters- Cabibbo angle $\lambda \approx 0.22$, A, ρ and η :

$$\mathbf{V}_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - 1/2\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - 1/2\lambda^2 & A\lambda^3 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \quad (1)$$

In the complex plane the point (ρ, η) is a vertex of the unitarity triangle and describes the CP- violation in SM. The unitarity triangle is constructed from the following unitarity condition of V_{CKM} : $V_{ub}^* + V_{cd} \approx A\lambda^3$.

Recently, the interest in the CP-violation problem was excited again due to the data on the search for the direct CP-violation effects in neutral K-mesons [1, 2] :

$$\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (7.4 \pm 6) \times 10^{-4}, \quad (2)$$

$$\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (23 \pm 7) \times 10^{-4}, \quad (3)$$

The major contribution to the CP-violation parameters ε_K and ε'_K (K^0 -decays), as well as to the $B_d^0 - \bar{B}_d^0$ mixing parameter $x_d = \frac{\Delta m(B_d)}{\Gamma(B_d)}$ is due to the large t-quark mass contribution. The same statement holds also for some amplitudes of K- and B-meson rare decays. The CDF collaboration gives the following region for the top quark mass: $m_t = 174 \pm 25$ GeV [3]. The complete fit which is based on the low energy data as well as the latest LEP and SLC data and comparing with the mass indicated by CDF measurements gives $m_t = 162 \pm 9$ GeV [4].

The combined analysis of the data on the $B_d^0 - \bar{B}_d^0$ mixing parameter $x_d = 0.67 \pm 0.10$ [5] and $\varepsilon_K = (2.26 \pm 0.02) \times 10^{-3} \exp(i43.7^\circ)$ indicated that the top quark mass should lie in the range of $(135 \pm 35)\text{GeV}$, although a very massive top quark in the range of $m_t \sim 200\text{GeV}$ is not excluded, either [6]. Note that for $m_t \approx O(100\text{GeV})$ the SM predicted $\left(\frac{\varepsilon'}{\varepsilon}\right)_K \approx (1.0 \pm 0.5) \times 10^{-3}$, and the value of $\delta_{13} = \arg V_{ub}^*$ ($V_{ub} = s_{13} \times \exp(-i\delta_{13})$) in the

second quadrant ($\delta_{13} > \frac{\pi}{2}$) of the unitary triangle is favored. For $m_t \sim 200 \text{ GeV}$ one gets the superweak- like behaviour predicted in the SM, i.e. $\left(\frac{\varepsilon'}{\varepsilon}\right)$ is close to zero. In this case the value of δ_{13} is likely to be in the first quadrant ($0 < \delta_{13} < \frac{\pi}{2}$).

It is worthwhile to note that the above conclusions depend strongly on the values of the hadronic matrix elements $\langle Q_6 \rangle, f_K, f_B$, as well as on the KM mixing angles s_{ij} . For example, the determination of $V_{cb} = s_{23} = 0.046 \pm 0.006$ from the $\Gamma(b \rightarrow c)$ decay rate and the determination of the ratio $q = \left|\frac{V_{ub}}{V_{cb}}\right|$ by ARGUS and CLEO lead to the model dependent results

$$0.07 \leq \left(q = \frac{s_{13}}{s_{23}} = \lambda \sqrt{\rho^2 + \eta^2}\right) \leq 0.12, \quad (4)$$

where the parameter $A \approx 0.8 - 0.9$ is defined from the dominant decays of B- hadrons ($V_{cb} = A\lambda^2$).

On the other hand, the unitarity triangle [7] constraint yields for

$|V_{td}|$, that $|V_{td}| = |s_{23}(\lambda - q \times \exp(i\delta_{13}))| = (0.0035 - 0.0200)$, ($\lambda = \sin \theta_C$). The value of V_{td} depends crucially on the δ_{13} phase. For the small values of this matrix element, δ_{13} tends to be in the first quadrant, whereas the experimental values for $\varepsilon_K, \frac{\varepsilon'_K}{\varepsilon_K}, x_d$ favor larger m_t values.

Another interesting possibility to check the sign of $\cos \delta_{13}$ comes from the experimental observation of the $B_s^0 - \bar{B}_s^0$ mixing. Due to the following relation between the parameters of the $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixings:

$$\frac{x_s}{x_d} = \frac{|V_{ts}|^2}{|V_{td}|^2} = \frac{1}{s_{12}^2 - 2qs_{12}\cos\delta_{13} + q^2} = \frac{1}{\lambda^2[(1-\rho)^2 + \eta^2]} \quad (5)$$

for $\delta_{13} \approx \pi$ we get $\frac{x_s}{x_d} = 10 - 14$ and $x_s = 7 - 11$, whereas for $\delta_{13} \approx 0$ we get $\frac{x_s}{x_d} = 36 - 100$ and $x_s = 27 - 70$.

The case of the symmetric form for the CKM-matrix [8], which leads to $V_{td} = V_{ub}$, $V_{ts} = V_{cb}$ and $V_{td} = \frac{1}{2}V_{us}V_{cb} = \frac{1}{2}V_{cd}V_{ts}$ ($q \simeq 0.1$ in this case), corresponds to δ_{13} -phase $\simeq 0$, $x_s/x_d \simeq 100$ and $x_s \simeq 70$.

As seen from the above discussion one can see that in the quark sector of the SM there are several experimental quantities, which are sensitive to the values of parameters $m_t, \delta^{KM}, B_d, \dots$. Therefore, we need additional experimental information to prove the validity of the SM with three generations and to get convinced that there are no additional contributions to the amplitudes of flavour changing rare processes due to new hypothetical forces beyond the SM.

1.2 Quark and lepton mass origin - mass ansatzes and quark mixing

The main drawbacks of SM now are going from our non-understanding the generation problem, their mixing and hierarchy of quark and lepton mass spectra. For example, for

quark masses $\mu \approx 1\text{GeV}$ we can get approximately the following relations [9]:

$$\begin{aligned} m_{i_k} &\approx (q_H^u)^{2k} m_0, \quad k = 0, 1, 2; \quad i_0 = u, i_1 = c, i_2 = t, \\ m_{i_k} &\approx (q_H^d)^{2k} m_0, \quad k = 0, 1, 2; \quad i_0 = d, i_1 = s, i_2 = b, \end{aligned} \quad (6)$$

where $q_H^u \approx (q_H^d)^2$, $q_H^d \approx 4 - 5 \approx 1/\lambda$ and $\lambda \approx \sin \theta_C$.

Here we used the conventional ratios of the "running" quark masses [10]

$$\begin{aligned} m_d/m_s &= 0.051 \pm 0.004, \quad m_u/m_c = 0.0038 \pm 0.0012, \quad m_s/m_b = 0.033 \pm 0.011, \\ m_c(\mu = 1\text{GeV}) &= (1.35 \pm 0.05)\text{GeV} \quad \text{and} \quad m_t^{phys} \approx 0.6m_t(\mu = 1\text{GeV}). \end{aligned} \quad (7)$$

This phenomenological formula (6) predicts the following value for the t -quark mass:

$$m_t^{phys} \approx 180 - 200\text{GeV}. \quad (8)$$

In SM these mass matrices and mixing come from the Yukawa sector :

$$L_Y = QY_u\bar{q}_u h^* + QY_d\bar{q}_d h + LY_e\bar{l}_e h + H.C., \quad (9)$$

where Q_i and L_i are three quark and lepton isodoublets, q_{u_i}, q_{d_i} and e_i are three right-handed antiquark and antilepton isosinglets, respectively. h is the ordinary Higgs doublet. In SM the 3×3 - family Yukawa matrices, $(Y_u)_{ij}$ and $(Y_d)_{ij}$ have no any particular symmetry. Therefore it is necessary to reach some additional mechanisms or symmetries beyond the SM which could diminish the number of the independent parameters in Yukawa sector L_Y . These new structures can be used for the determination of the mass hierarchy and family mixing.

To understand the generation mixing origin and fermion mass hierarchy several models beyond the SM suggest special forms for the mass matrix of "up" and "down" quarks (Fritzsch ansatz, "improved" Fritzsch ansatz, "Democratic" ansatz, etc.[11]). These mass matrices have less than ten independent parameters or they could have some matrix elements equal to zero ("texture zeroes")[12]. This allows us to determine the diagonalizing matrix U_L and D_L in terms of quark masses:

$$Y_d^{diag} = D_L Y_d D_R^+, \quad Y_u^{diag} = U_L Y_u U_R^+. \quad (10)$$

For simplicity it has been taken the symmetric form of Yukawa matrices, therefore: $D_L = D_R^*$, $U_L = U_R^*$. These ansatzes or zero "textures" could be checked experimentally in predictions for the mixing angles of the CKM matrix: $V_{CKM} = U_L D_L^+$. For example, it can be considered the next approximate form at M_X for the symmetric "texture", using in paper [12]:

$$\mathbf{Y}_u = \begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & 0 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix} \quad \mathbf{Y}_d = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 2\lambda^3 \\ 0 & 2\lambda^3 & \end{pmatrix}. \quad (11)$$

Given these conditions it is possible to evolve down to low energies via the renormalization group equations all quantities including the matrix elements of Yukawa couplings $Y_{u,d}$, the values of the quark masses and the CKM matrix elements [12]. Also, using these relations we may compute U_L (or D_L) in terms of CKM matrix and/or of quark masses.

In GUT extensions of the SM with embedding the family gauge symmetry Yukawa matrices can acquire particular symmetry or an ansatz, depending on the Higgs multiplets to which they couple. The family gauge symmetry could help us to study by independent way the origin of the up- (U) and down- (D) quark mixing matrices and consequently the structure of the CKM matrix $V_{CKM} = UD^+$. The possibility a low energy breaking scale gives us a chance due to the local gauge family symmetry to define the quantum numbers of quarks and leptons and thus establishes a link between them in families. For considering an mass fermion ansatz in the extensions of SM there could exist the following types of the $SU(3) \times SU(2_L)$ Higgs multiplets: $(1,2), (3,1), (8,1), (3,2), (8,2), (1,1), \dots$, which could exist in spectra of the String Models.

In the framework of the rank eight Grand Unified String Theories we will consider an extension of SM due to local family gauge symmetry, $G_H = SU(3)_H, SU(3)_H \times U(1)_H$ models and its developments and the possible Higgs sector in them. Thus, for understanding the quark mass spectra and the difference between the origins of the up- (or down) quark and charged lepton mass matrices in GUSTs we have to study the Higgs content of the model, which we must use from one side for breaking the GUT-, Quark-Lepton -, $G_H = SU(3)_H, \dots, SU(2)_L \times U(1)$ - symmetries and from another side for Yukawa matrix constructions. The vital question arising here is the nature of the ν mass.

1.3 The possible ways of $E(8)$ - Grand Unified String Theories leading to the $N_G = 3$ or $N_G = 3 + 1$ families

For a couple of years superstring theories, and particularly the heterotic string theory, have provided an efficient way to construct the Grand Unified Superstring Theories (*GUST*) of all known interactions, despite the fact that it is still difficult to construct unique and fully realistic low energy models resulting after decoupling of massive string modes. This is because it is only in 10-dimensional space-time that there exist just two consistent (invariant under reparametrization, superconformal, modular, Lorentz and SUSY transformations) theories with the gauge symmetries $E(8) \times E(8)$ or $spin(32)/Z_2$ [13, 14] which after compactification of the six extra space coordinates (into the Calabi-Yau [15, 16] manifolds or into the orbifolds) can be used for constructing GUSTs. Unfortunately, the process of compactification to four dimensions is not unique and the number of possible low energy models is very large. On the other hand, starting the construction of the theory directly

in 4-dimensional space-time requires including a considerable number of free bosons or fermions into the internal string sector of the heterotic superstring [17, 18, 19, 20]. This leads to as large internal symmetry group such as e.g. rank 22 group. The way of breaking this primordial symmetry is again not unique and leads to a huge number of possible models, each of them giving different low energy predictions.

On the other side, because of the presence of the affine Kac-Moody algebra (KMA) \hat{g} (which is a 2-dimensional manifestation of gauge symmetries of the string itself) on the world sheet, string constructions yield definite predictions as to what representation of the symmetry group can be used for low energy models building [21, 22]. Therefore the following long-standing questions have a chance to be answered in this kind of unification schemes:

1. How are the chiral matter fermions assigned to the multiplets of the unifying group?
2. How is the GUT gauge symmetry breaking realized?
3. What is the origin and the form of the fermion mass matrices?

The first of these problems is, of course, closely connected with the quantization of the electromagnetic charge of matter fields. In addition, string constructions can shed some light on the questions about the number of generation and possible existence of mirror fermions which remain unanswered in conventional GUTs [23].

There are not so many GUSTs describing the observable sector of String Models. It is well known the SM gauge group, the Pati-Salam ($SU(4) \times SU(2) \times SU(2)$) gauge group, the flipped $SU(5)$ gauge group and $SO(10)$ gauge group, which includes flipped $SU(5)$ [20].

There are good physical reasons for including the horizontal $SU(3)_H$ group into the unification scheme. Firstly, this group naturally accommodates three fermion families presently observed (explaining their origin) and, secondly, can provide correct and economical description of the fermion mass spectrum and mixing without invoking high dimensional representation of conventional $SU(5)$, $SO(10)$ or $E(6)$ gauge groups. Construction of a string model (GUST) containing the horizontal gauge symmetry provides additional, strong motivation to this idea. Moreover, the fact that in GUSTs high dimensional representations are forbidden by the KMA is a very welcome feature in this context.

All this leads us naturally to consider possible forms for horizontal symmetry G_H , and G_H quantum number assignments for quarks (anti-quarks) and leptons (anti-leptons) which can be realized within GUST's framework. To include the horizontal interactions with three known generations in the ordinary GUST it is natural to consider rank eight gauge symmetry. We can consider $SO(16)$ (or $E(6) \times SU(3)$) which is the maximal subgroup of $E(8)$ and which contains the rank eight subgroup $SO(10) \times (U(1) \times SU(3))_H$ [24]. We will be, therefore, concerned with the following chains (see Fig. 1):

Figure 1: The possible ways of E(8) gauge symmetry breaking leading to the 3+1 or 3 generations.

$$\begin{array}{ccc}
E(8) & \xrightarrow{248 \longrightarrow 120 \oplus 128} & SO(16) \\
\downarrow \begin{array}{l} 248 \longrightarrow \\ (78, 1) \oplus \\ (1, 8) \oplus \\ (27, 3) \oplus \\ (27, \bar{3}) \end{array} & & \downarrow \begin{array}{l} 120 \longrightarrow (45, 1)^0 \oplus (1, 8)^0 \oplus \\ (1, 1)^0 \oplus (10, 3)^2 \oplus (10, \bar{3})^{-2} \oplus \\ (1, 3)^{-4} \oplus (1, 3)^{+4} \\ 128 \longrightarrow (16, 3)^{-1} \oplus (\bar{16}, \bar{3})^{+1} \oplus \\ (16, 1)^{+3} \oplus (\bar{16}, 1)^{-3} \end{array} \\
\begin{array}{l} (78, 1) \longrightarrow (45, 1)^0 \oplus (1, 1)^0 \oplus (16, 1)^{+3} \oplus (\bar{16}, 1)^{-3} \\ (27, 3) \longrightarrow (16, 3)^{-1} \oplus (10, 3)^{+2} \oplus (1, 3)^{-4} \\ (27, \bar{3}) \longrightarrow (\bar{16}, \bar{3})^{+1} \oplus (10, \bar{3})^{-2} \oplus (1, \bar{3})^{+4} \end{array} & & \\
E(6) \times SU(3)_H & \longrightarrow & SO(10) \times SU(3)_H \times U(1)_H \\
\downarrow \begin{array}{l} 78 \longrightarrow \\ (8, 1, 1) \oplus \\ (1, 8, 1) \oplus \\ (1, 1, 8) \oplus \\ (3, 3, 3) \oplus \\ (\bar{3}, \bar{3}, \bar{3}) \\ 27 \longrightarrow \\ (3, \bar{3}, 1) \oplus \\ (1, 3, \bar{3}) \oplus \\ (\bar{3}, 1, 3) \end{array} & & \downarrow \begin{array}{l} 45 \longrightarrow (24, 1) \oplus (1, 1) \oplus (10, 1) \oplus (\bar{10}, 1) \\ 16 \longrightarrow (1)_{+5/2} \oplus (\bar{5})_{-3/2} \oplus (10)_{+1/2} \\ \bar{16} \longrightarrow (\bar{1})_{-5/2} \oplus (5)_{+3/2} \oplus (\bar{10})_{-1/2} \end{array} \\
SU(3)^{\otimes 4} & & SU(5) \times U(1) \times SU(3)_H \times U(1)_H \\
& & N_g = 3, N_g = 3 + 1
\end{array}$$

$$\begin{aligned}
E(8) &\longrightarrow SO(16) \longrightarrow \frac{SO(10) \times (U(1) \times SU(3))_H}{\longrightarrow} \\
&\longrightarrow SU(5) \times U(1)_{Y_5} \times (SU(3) \times U(1))_H
\end{aligned}$$

or

$$E(8) \longrightarrow E(6) \times SU(3) \longrightarrow (SU(3))^{\times 4}.$$

According to this scheme one can get $SU(3)_H \times U(1)_H$ gauge family symmetry with $N_g = 3 + 1$ (there are also other possibilities as eg. $E(6) \times SU(3)_H \subset E(8)$ $N_g = 3$ generations can be obtained due to the second way of $E(8)$ gauge symmetry breaking via $E(6) \times SU(3)_H$, see Fig.1), where the possible, additional, fourth massive matter superfield could appear from $\underline{78}$ as a singlet of $SU(3)_H$ and transforms as $\underline{16}$ under the $SO(10)$ group.

In this note starting from the rank 16 grand unified gauge group (which is the minimal rank allowed in strings [25, 26]) of the form $G \times G$ and making use of the KMA which select the possible gauge group representations we construct the string model based on the diagonal subgroup $G^{symm} \subset G \times G \subset SO(16) \times SO(16) (\subset E_8 \times E_8)$ [25]. We discuss and consider $G^{symm} = SU(5) \times U(1) \times (SU(3) \times U(1))_H \subset SO(16)$ where the factor $(SU(3) \times U(1))_H$ is interpreted as the horizontal gauge family symmetry. We explain how the unifying gauge symmetry can be broken down to the Standard Model group. Furthermore, the horizontal interaction predicted in our model can give an alternative description of the fermion mass matrices without invoking high dimensional Higgs representations. In contrast with other GUST constructions, our model does not contain particles with exotic fractional electric charges [27, 25]. This important virtue of the model is due to the symmetric construction of the electromagnetic charge Q_{em} from Q^I and Q^{II} – the two electric charges of each of the $U(5)$ groups [25]:

$$Q_{em} = Q^{II} \oplus Q^I. \quad (12)$$

We consider the possible forms of the $G_H = SU(3)_H, SU(3)_H \times U(1), G_{HL} \times G_{HR} \dots$ - gauge family symmetries in the framework of Grand Unification Superstring Approach. Also we will study the matter spectrum of these GUST, the possible Higgs sectors. The form of the Higgs sector it is very important for GUST-, G_H - and SM - gauge symmetries breaking and for constructing Yukawa couplings.

1.4 Towards a low energy gauge family symmetry "exactly solvable". ("Bootstrap" models.)

The underlying analysis for this family symmetry breaking scale is lying on the modern experimental probability limitations for the typical rare flavour- changing processes. The estimates for the family symmetry breaking scale have certain regularities depending on the particular symmetry breaking schemes and generation mixing mechanisms (different anzatzes for quark and lepton mass matrices with 3_H or $3_H + 1_H$ generations have been discussed). As noted, the current understanding of quark and lepton mass spectra leaves room for the existence of an unusually low mass breaking scale of non-abelian gauge $SU(3)_H$ or $(SU(3) \otimes U(1))_H$ family symmetry $\sim some TeV$. Some independent experiments for verifying the relevant hypotheses can be considered: light (π, K), heavy (B, D) - meson and charged lepton flavour changing rare decays [28, 29, 30, 9], family symmetry violation effects in e^+e^- - and pp - collider experiments (LEP, FNAL, LHC).

The introductions into the model the Higgs fields which are transformed under the $SU(3)_H \times SU(2)_L$ symmetry, like as $H^a = (\underline{3}, \underline{1})$ (or $H_p^a = (\underline{3}, \underline{2})$, p=1,2) and $X^i = (\underline{3}, \underline{1})$ (or $X_p^i = (\underline{3}, \underline{2})$, p=1,2) give the next contribution to the family gauge boson mass matrix:

$$(M_H^2)_{\underline{3}}^{ab} = g_H^2 \sum_{d=1}^8 f^{adc} f^{bdc'} \langle H^c \rangle \langle H^{c'} \rangle, \quad (13)$$

$$(M_H^2)_{\underline{3}}^{ab} = g_H^2 \sum_{k=1}^3 \frac{\lambda_{ik}^a}{2} \frac{\lambda_{kj}^b}{2} < X^i > < X^j >^*, \quad (14)$$

The lowest bound on M_H can be obtained from the analysis of the branching ratios of μ, π, K, D, B, \dots rare decays ($\text{Br} \geq 10^{-15-17}$).

In this paper we will investigate the samples of different scenarios of $SU(3)_H$ - breakings up to the $SU(2)_H \times U(1)_{3H}$, $U(1)_{3H} \times U(1)_{8H}$ and $U(1)_{8H}$ - subgroups, as well as the mechanism of the complete breaking of the base group $SU(3)_H$ - [9]. We will try to realize this program conserving SUSY on the scales where the relevant gauge symmetry is broken. In the framework of these versions of the gauge symmetry breaking, we will search for the spectra of horizontal gauge bosons and gauginos and calculate the amplitudes of some typical rare processes. Theoretical estimates for the branching ratios of some rare processes obtained from these calculations will be compared with the experimental data on the corresponding values. Further on we will get some bounds on the masses of H_μ - bosons and the appropriate H -gauginos. Of particular interest is the case of the $SU(3)_H$ -group which breaks completely on the scale M_{H_0} . We calculate the splitting of eight H -boson masses in a model dependent fashion. This splitting, depending on the quark mass spectrum, allows us to reduce considerably the predictive ambiguity of the model -"almost exactly solvable model".

We assume that when the $SU(3)_H$ -gauge symmetry of quark- lepton generations is violated, all the 8 gauge bosons acquire in the eigenspectrum of horizontal interactions the same mass equal to M_{H_0} . The such breaking is not difficult to get by, say, introducing the Higgs fields transforming in accordance with the triplet representation of the $SU(3)_H$ group. These fields are singlet under the Standard Model symmetries : $(z \in (3, 1, 1, 0)$ and $\bar{z} \in (\bar{3}, 1, 1, 0)$, $< \bar{z}^{i\alpha} >_0 = \delta^{i\alpha} V$, $< z_i^\alpha >_0 = \delta_i^\alpha V$, $i, \alpha = 1, 2, 3$, where $V = M_{H_0}$). We understand that here we need in more beautiful way to break this symmetry like by dynamical way. But at this stage it is very important now to establish a link between the spectra masses of the horizontal gauge bosons and of till known now the matter fermion heavy particles like t- quark. The degeneracy in the masses of 8 gauge horizontal vector bosons is eliminated by using the VEV's of the Higgs fields violating the electroweak symmetry and determining the mass matrix of up- and down- quarks (leptons). Thus, in the set of the Higgs fields (see Table 11), with $H(8, 2)$, $h(8, 2)$, $Y(\bar{3}, 2)$, $X(3, 2)$, $\kappa_{1,2}(1, 2)$ violates the $SU(2) \times U(1)$ symmetry and determines the mass matrix of up-and down-quarks. On the other hand, in order to calculate the splitting between the masses of horizontal gauge bosons, one has to take into account the VEV's of these two sets of the Higgs fields.

Now we can come to constructing the horizontal gauge boson mass matrix M_{ab}^2 (a,b=1,2,...,8):

$$(M_H^2)_{ab} = M_{H_0}^2 \delta_{ab} + (\Delta M_d^2)_{ab} + (\Delta M_u^2)_{ab}. \quad (15)$$

Here $(\Delta M_d^2)_{ab}$ and $(\Delta M_u^2)_{ab}$ are the "known" functions of heavy fermions, $(\Delta M_{u,d}^2)_{ab} = F_{ab}(m_t, m - b, \dots)$, which, mainly, get the contributions due to the vacuum expectations

of the Higgs bosons that were used for construction of the mass matrix ansatzes for d-(u-) quarks.

For example, for the case $N_g = \underline{3} + \underline{1}$ families with Fritzsch ansatz for quark mass matrices and using $SU(3_H) \times SU(2)$ Higgs fields, (8, 2), [9], we can write down some rough equalities between the masses of horizontal gauge bosons:

$$\begin{aligned}
M_{H_1}^2 &\approx M_{H_2}^2 \approx M_{H_3}^2 \approx M_{H_0}^2 + \frac{g_H^2}{4} \left[\frac{1}{\lambda^2} \frac{m_c m_t}{1 - m_t/m_{t'}} \right] + \dots, \\
M_{H_4}^2 &\approx M_{H_5}^2 \approx M_{H_6}^2 \approx M_{H_7}^2 \approx M_{H_0}^2 + \frac{g_H^2}{4} \left[\frac{1}{\tilde{\lambda}^2} m_t m_{t'} \right] + \dots, \\
M_{H_8}^2 &\approx M_{H_0}^2 + \frac{g_H^2}{3} \left[\frac{1}{\tilde{\lambda}^2} m_t m_{t'} \right] + \dots,
\end{aligned} \tag{16}$$

where λ and $\tilde{\lambda}$ are Yukawa couplings.

We are interested in the dependence of the unitary compensation for the contributions of horizontal forces to rare processes [9] on different versions of the $SU(3)_H$ - symmetry breaking. The investigation of this dependence allows, first, to understand how low the horizontal symmetry breaking scale M_H may be, and, second, how this scale is determined by a particular choice of a mass matrix ansatz both for quarks and leptons.

We would like to consider of a possible existing of the local family symmetry with a low energy symmetry breaking scale, i.e. the existence of rather light H-bosons: $m_H \geq (1-10)TeV$ [9]. We will analyze, in the framework of the "minimal" horizontal supersymmetric gauge model, the possibilities to obtain a satisfactory hierarchy for quark masses and to connect it with the splitting of horizontal gauge boson masses. We expect that due to this approach the horizontal model will become more definite since it will allow to study the amplitudes of rare processes and the CP-violation mechanism more thoroughly. In this way we hope to get a deeper insight into the nature of interdependence between the generation mixing mechanism and the local horizontal symmetry breaking scale.

1.5 Estimates on the horizontal coupling constant and the scale of unification.

Really, the estimates on the M_{H_0} - scale depend on the value of the family gauge coupling. These estimates can be taken in GUST using the string scale

$$M_U \approx 0,73 g_{string} \times 10^{18} GeV \tag{17}$$

and the renormalization group equations (RGE) for the gauge couplings, α_{em} , α_3 , α_2 , to the low energies : $\alpha_{em}(M_Z) \approx 1/128$, $\alpha_3(M_Z) \approx 0,11$, $\sin^2 \theta_W(M_Z) \approx 0.233$. The string unification scale could be contrasted with the $SU(3^c) \times SU(2) \times U(1)$ naive unification scale, $M_U \approx 10^{16} GeV$, obtained by running the SM particles and their SUSY-partners to high energies. The simplest solution to this problem is the introduction in the spectrum

of new heavy particles with SM quantum numbers, which can be exist in string spectra [20]d.

However there are some other ways to explain the difference between scales of string (M_{SU}) and ordinary (M_U) unifications. Thus if one uses the breaking scheme $G \times G \rightarrow G^{sym}$ (where $G = U(5) \times U(3)_H \subset E_8$) described above, then unification scale $M_U \sim 10^{16}$ GeV is the scale of breaking the $G \times G$ group, and string unification do supply the equality of coupling constant $G \times G$ on the string scale $M_{SU} \sim 10^{18}$ GeV. In addition if there is a symmetry between representations of two groups G then

$$g_{sym}(M_U) = \frac{1}{\sqrt{2}} g_G(M_U),$$

but in absence of symmetry the relation is more complicated. Thus in this scheme knowing of scales M_{SU} and M_U gives us a principal possibility to trace the evolution of coupling constant of the original group $G \times G$ to the low energy and estimate the value of horizontal gauge constant g_{3H} .

The coincidence of $\sin^2 \theta_W$ with experiment will show how realistic this model is. In our scheme, where $G_{sym} = SU(5) \times U(1) \times U(3)_H$ the following equation holds:

$$\sin^2 \theta_W(M_U) = \frac{15 k^2}{16 k^2 + 24} \Big|_{k^2=1} = \frac{3}{8} \quad (18)$$

The relation between $SU(5)_{sym} \times U(1)_{sym}$ -constants $k = g_1/g_5$ on the scale of M_U is defined by set of representations of $G^I \times G^{II}$ group. The analysis of RG-equations under the M_U scale allows to state that horizontal coupling constant g_{3H} does not exceed electro-weak one g_2 .

In special case [9] the difference between the $\alpha_2(\mu)$ and $\alpha_{3H}(\mu)$ is mainly due to the particular choice of the Higgs fields leading to the breaking of the electroweak and horizontal symmetries, respectively:

$$\begin{aligned} \frac{1}{\alpha_{3H}(\mu)} &= \frac{1}{\alpha_2(\mu)} + \frac{b_{3H} - b_2}{2\pi} \ln \frac{M_X}{\mu} \\ &= \frac{1}{\alpha_2(\mu)} + \frac{-1 + 1/2 n_3 + 3 n_8 - 1/2 n_2}{2\pi} \ln \frac{M_X}{\mu}, \end{aligned} \quad (19)$$

where n_2 and n_3, n_8 denote the nubmer of the Higgs $SU(2)$ - doublet and $SU(3)_H$ triplet and octet, respectively. Therefore, for instance, when one takes into account the fields $\Phi(8_H, 1_L), H(8_H, 2_L), h(8_H, 2_L)$, the difference between $\alpha_{3H}(\mu)$ and $\alpha_2(\mu)$ is expressed by the formula:

$$\frac{1}{\alpha_{3H}(\mu)} - \frac{1}{\alpha_2(\mu)} = \frac{1}{2\pi} \ln \frac{M_X}{\mu} \quad (20)$$

whence $\alpha_{3H}(\mu) \leq \alpha_2(\mu)$.

1.6 The N=1 SUSY character of the $SU(3_H)$ - gauge family symmetry

We will consider the supersymmetric version of the Standard Model extended by the family (horizontal) gauge symmetry (and if one will need we will also extend this model by the $G_R = SU(2)_R$ Right-hand gauge group). The supersymmetric Lagrangian of strong, electroweak and horizontal interactions, based on the $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3)_H \dots$ (where the G_R -gauge group and the Abelian gauge factor $U(1)_H$ also can be taken into consideration), has the general form:

$$\begin{aligned}
\mathcal{L} = & \int d^2\theta \, Tr (W^k W^k) \\
& + \int d^4\theta \, S_I^+ e^{\sum_k 2g_k \hat{V}_k} S_I \\
& + \int d^4\theta \, Tr(\Phi^+ e^{2g_H \hat{V}_H} \Phi e^{-2g_H \hat{V}_H}) \\
& + \int d^4\theta \, Tr(H_y^+ e^{2g_2 \hat{V}_2 + y 2g_1 \hat{V}_1} e^{2g_H \hat{V}_H} H_y e^{-2g_H \hat{V}_H}) \\
& + \left(\int d^2\theta \, P(S_i, \Phi, H_y, \eta, \xi, \dots) + h.c. \right)
\end{aligned} \tag{21}$$

In formula (21) the index k runs over all the gauge groups: $SU(3)_C, SU(2)_L, U(1)_Y, SU(3)_H$, $\hat{V} = \mathbf{T}^a V^a$, where V^a are the real vector superfields, and \mathbf{T}^a are the generators of the $SU(3)_C, SU(2)_L, U(1)_Y, SU(3)_H$ -groups; S_I are left-chiral superfields from fundamental representations, and $I = i, 1, 2$; $S_i = Q, u^c, d^c$, L, e^c, ν^c - are matter superfields, $S_1 = \eta, S_2 = \xi$ - are Higgs fundamental superfields; the Higgs left chiral superfield Φ is transformed according to the adjoint representation of the $SU(3)_H$ -group, the Higgs left chiral superfields H_y : $H_{Y=+\frac{1}{2}} = H$, $H_{Y=-\frac{1}{2}} = h$ are transformed nontrivially under the horizontal $SU(3)_H$ - and electroweak $SU(2)_L$ - symmetries (see Table 11). P in formula (21) is a superpotential to be specified below. To construct it, we use the internal $U(1)_R$ -symmetry which is habitual for a simple N=1 supersymmetry.

In models with a global supersymmetry it is impossible simultaneously to have a SUSY breaking and a vanishing cosmological term. The reason is the semipositive definition of the scalar potential in the rigid supersymmetry approach (in particular, in the case of a broken $SUSY$ we have $V_{min} > 0$). The problem of supersymmetry breaking, with the cosmological term $\Lambda = 0$ vanishing, is solved in the framework of the $N = 1$ $SUGRA$ models. This may be done under an appropriate choice of the Kaehler potential, in particular, in the frames of "mini-maxi"- or "maxi" type models [31]. In such approaches, the spontaneous breaking of the local $SUSY$ is due to the possibility to get nonvanishing VEV s for the scalar fields from the "hidden" sector of $SUGRA$ [31]. The appearance in the observable sector of the so-called soft breaking terms comes as a consequence of this effect.

In the "flat" limit, i.e. neglecting gravity, one is left with lagrangian (21) and soft

SUSY breaking terms, which on the scales $\mu \ll M_{Pl}$ have the form:

$$\begin{aligned}
\mathcal{L}_{SB} = & \frac{1}{2} \sum_i m_i^2 |\phi_i|^2 + \frac{1}{2} m_1^2 Tr|h|^2 + \frac{1}{2} m_2^2 Tr|H|^2 + \\
& + \frac{1}{2} \mu_1^2 |\eta|^2 + \frac{1}{2} \mu_2^2 |\xi|^2 + \frac{1}{2} M^2 Tr|\Phi|^2 + \\
& + \frac{1}{2} \sum_k M_k \lambda_k^a \lambda_k^a + h.c. + \text{trilinear terms},
\end{aligned} \tag{22}$$

where i runs over all the scalar matter fields $\tilde{Q}, \tilde{u}^c, \tilde{d}^c, \tilde{L}, \tilde{e}^c, \tilde{\nu}^c$ and k - does over all the gauge groups: $SU(3)_H, SU(3)_C, SU(2)_L, U(1)_Y$. At the energies close to the Plank scale all the masses, as well as the gauge coupling, are correspondingly equal (this is true if the analytic kinetic function satisfies $f_{\alpha\beta} \sim \delta_{\alpha\beta}$) [31], but at low energies they have different values depending on the corresponding renormgroup equation (RGE). The squares of some masses may be negative, which permits the spontaneous gauge symmetry breaking.

Considering the SUSY version of the $SU(3)_H$ -model, it is natural to ask: why do we need to supersymmetrize the model? Proceeding from our present-day knowledge of the nature of supersymmetry [31, 32], the answer will be:

(a) First, it is necessary to preserve the hierarchy of the scales: $M_{EW} < M_{SUSY} < M_H < \dots < M_{GUT}$. Breaking the horizontal gauge symmetry, one has to preserve SUSY on that scale. Another sample of hierarchy to be considered is : $M_{EW} < M_{SUSY} \sim M_H$. In this case, the scale M_H should be rather low ($M_H \leq$ a few TeV).

(b) To use the SUSY $U(1)_R$ degrees of freedom for constructing the superpotential and forbidding undesired Yukawa couplings.

(c) Super-Higgs mechanism - it is possible to describe Higgs bosons by means of massive gauge superfields [32].

(d) To connect the vector- like character of the $SU(3)_H$ - gauge horizontal model and $N = 2$ SUSY.

1.7 The superweak-like source of CP- violation, the Baryon stability and neutrino mass problems.

The existence of horizontal interactions (21) might be closely connected with the CP-violation problem. This interaction is described by the relevant part of the SUSY $SU(3)_H$ -Lagrangian and has the form

$$\mathcal{L}_H = g_H \bar{\psi}_d \Gamma_\mu (D P_d \frac{\Lambda^a}{2} P_d^* D^T) \psi_d O_{ab} Z_\mu^b. \tag{23}$$

Here we have (a,b=1,2,...,8). The matrix O_{ab} determines the relationship between the bare, H_μ^b , and physical, Z_μ^b , gauge fields and is calculated for the mass matrix $(M_H^2)_{ab}$ diagonalized; $\psi_d = (\psi_d, \psi_s, \psi_b)$; g_H is the gauge coupling of the $SU(3)_H$ group.

Here noteworthy are the following two points: The appearance of the phase in the CKM mixing matrix may be due to new dynamics working at short distances ($r \ll \frac{1}{M_W}$). Horizontal forces may be the source of this new dynamics [9]. Using this approach, we might have the CP- violation effects- both due to electroweak and horizontal interactions.

(b) The CP is conserved in the electroweak sector ($\delta^{KM} = 0$), and its breaking is provided by the structure of the horizontal interactions. Let us think of the situation when $\delta^{KM} = 0$. In the SM, such a case might be realized just accidentally. The vanishing phase of the electroweak sector ($\delta^{KM} = 0$) might arise spontaneously due to some additional symmetry. Again, such a situation might occur within the horizontal extension of the electroweak model.

In particular, this model gives rise to a rather natural mechanism of superweak - like CP-violation due to the ($CP = -1$) part of the effective Lagrangian of horizontal interactions- ($\epsilon'/\epsilon)_K \leq 10^{-4}$. That part of \mathcal{L}_{eff} includes the product of the $SU(3)_H$ - currents $I_{\mu i} I_{\mu j}$ ($i=1,4,6,3,8$; $j=2,5,7$ or, vice versa, $i \longleftrightarrow j$) [9]. In the case of a vector-like $SU(3)_H$ - gauge model the CP- violation could be only due to the charge symmetry breaking.

In electroweak and horizontal interactions we might also have two CP- violating contributions to the amplitudes of B-meson decays. But it is possible to construct a scheme where CP- violation will occur only in the horizontal interactions. The last fact might lead to a very interesting CP -violation asymmetry $A_f(t)$ for the decays of neutral B_d^0 - and \bar{B}_d^0 - mesons to final hadron CP -eigenstates ,for example, to $f = (J/\Psi K_S^0)$ or $(\pi^+ \pi^-)$

$$A_f(t) \approx \sin(\Delta m_{B_d} t) \text{Im}\left(\frac{p}{q} \times \rho_f\right), \quad \rho_f = \frac{A(\bar{B} \rightarrow f)}{A(B \rightarrow f)}. \quad (24)$$

In the standard model with the Kobayashi- Maskawa mechanism of CP- violation, the time integrated asymmetry of B_d^0 - and \bar{B}_d^0 - meson decays to the $J/\Psi K_S^0$ - final state is:

$$A(J/\Psi K_S^0) \approx \eta_f \times \frac{x_d}{1+x_d^2} \times \sin 2\phi_3 = -\frac{x_d}{1+x_d^2} \times \frac{2\eta(1-\rho)}{(1-\rho)^2 + \eta^2},$$

where $\eta_f=-1$ for a CP-odd $J/\Psi K_S^0$ - final state; $\phi_3 = \arg V_{td}$ is one of the angles: ($\phi_i, i = 1, 2, 3$) of the unitary triangle. Let us compare this asymmetry with the analogous asymmetry of the B^0 and \bar{B}^0 -decays to the CP- even final state (π^+, π^-) , the latter being known to depend on the phase magnitudes of V_{ub} and V_{td} . Then:

$$A(\pi^+ \pi^-) \approx -\eta_f \times \frac{x_d}{1+x_d^2} \times \sin 2\phi_2 = -\frac{x_d}{1+x_d^2} \times \frac{2\eta[(\rho^2 + \eta^2) - \rho]}{[(1-\rho)^2 + \eta^2][\rho^2 + \eta^2]},$$

where $\phi_2 = \pi - \phi_1 - \phi_3$ and $\phi_1 = \arg V_{ub}^* = \delta_{13}$ ($\delta^{KM} = \phi_1 + \phi_3$).

The contributions of CP-violating horizontal interactions to the asymmetries for both B^0 -decays are identical but the signs differ.

The space-time structure of horizontal interactions depends on the $SU(3)_H$ quantum numbers of quark and lepton superfields and their C- conjugate superfields. One can obtain vector (axial)-like horizontal interactions as far as the G_H particle quantum numbers are conjugate (equal) to those of antiparticles. The question arising in these theories is how such horizontal interactions are related with strong and electroweak ones. All these interactions can be unified within one gauge group, which would allow to calculate the value of the coupling constant of horizontal interactions. Thus, an unification of horizontal, strong and electroweak interactions might rest on the GUTs $\tilde{G} \equiv G \times SU(3)_H$ (where, for example, $\tilde{G} \equiv E(8)$, $G \equiv SU(5)$, $SO(10)$ or E_6), which may be further broken down to $SU(3)_H \times SU(3)_C \times SU(2)_L \times U(1)_Y$. For including "vector"- like horizontal gauge symmetry into GUT we have to introduce "mirror" superfields. Speaking more definitely, if we want to construct GUTs of the $\tilde{G} \equiv G \times SU(3)_H$ type, each generation must encompass double G -matter supermultiplets, mutually conjugate under the $SU(3)_H$ -group. In this approach the first supermultiplet consists of the superfields f and $f_m^c \in \mathbf{3}_H$, while the second is constructed with the help of the supermultiplets f^c and $f_m \in \bar{\mathbf{3}}_H$. In this scheme, proton decays are only possible in the case of mixing between ordinary and "mirror" fermions. In its turn, this mixing must, in particular, be related with the $SU(3)_H$ -symmetry breaking.

The GUSTs spectra also predict the existing of the new neutral neutrino - like particles interacting with the matter only by "superweak"- like coupling. It is possible to estimate the masses of these particles, and, as will be shown further, some of them have to be light (superlight) to be observed in modern experiment.

2 Non-Abelian Gauge Family Symmetry in Grand Unified String Models

2.1 World-Sheet Kac-Moody Algebra And Main Features of Rank Eight GUST

2.1.1 The representations of Kac- Moody Algebra and Vertex Operators

Let's begin with a short review of the KMA results [21, 22]. In heterotic string the KMA is constructed by the operator product expansion (OPE) of the fields J^a of the conformal dimension $(0, 1)$:

$$J^a(z)J^b(w) = \frac{1}{z-w}k\delta^{ab} + \frac{1}{z-w}if^{abc}J^c + \dots \quad (25)$$

The structure constants f^{abc} for the group g are normilized so that

$$f^{acd}f^{bcd} = Q_\psi\delta^{ab} = \tilde{h}\psi^2\delta^{ab} \quad (26)$$

, where Q_ψ and ψ are the quadratic Casimir and the highest weight of the adjoint representation and \tilde{h} is the dual Coxeter number. The $\frac{\psi}{\psi^2}$ can be expanded as in integer linear

combination of the simple roots of g :

$$\frac{\psi}{\psi^2} = \sum_{i=1}^{\text{rank } g} m_i \alpha_i. \quad (27)$$

The dual Coxeter number can be expressed through the integers numbers m_i

$$\tilde{h} = 1 + \sum_{i=1}^{\text{rank } g} m_i \quad (28)$$

and for the simply laced groups (all roots are equal and $\psi^2 = 2$): A_n, D_n, E_6, E_7, E_8 they are equal $n + 1, 2n - 2, 12, 18$ and 30 , respectively.

The KMA \hat{g} allow to grade the representations R of the gauge group by a level number x (a non negative integer) and by a conformal weight $h(R)$. An irreducible representation of the affine algebra \hat{g} is characterized by the vacuum representation of the algebra g and the value of the central term k , which is connected with the level number by the relation $x = 2k/\psi^2$. The value of the level number of the KMA determines the possible highest weight unitary representation which are present in the spectrum, in the following way:

$$x = \frac{2k}{\psi^2} \geq \sum_{i=1}^{\text{rank } g} n_i m_i, \quad (29)$$

where the sets of non-negative integers $\{m_i = m_1, \dots, m_r\}$ and $\{n_i = n_1, \dots, n_r\}$ define the highest root and the highest weight of a representation R respectively [21, 22]:

$$\mu_0 = \sum_{i=1}^{\text{rank } g} n_i \alpha_i \quad (30)$$

In fact, the KMA on the level one is realized in the 4-dimensional heterotic superstring theories with free world sheet fermions which allows a complex fermion description [18, 19, 20]. One can obtain KMA on higher level working with real fermions using some tricks [33]. For these models the level of KMA coincides with the Dynkin index of representation M to which free fermions are assigned:

$$x = x_M = \frac{Q_M}{\psi^2} \frac{\dim M}{\dim g} \quad (31)$$

(Q_M is a quadratic Casimir eigenvalue of representation M) and equals one in cases when real fermions form vector representation M of $SO(2N)$, or when the world sheet fermions are complex and M is the fundamental representation of $U(N)$ [21, 22].

Thus, in strings with KMA on the level one realized on the world-sheet, only very restricted set of unitary representations can arise in the spectrum:

1. singlet and totally antisymmetric tensor representations of $SU(N)$ groups, for which $m_i = (1, \dots, 1)$;

2. singlet, vector and spinor representations of $SO(2N)$ groups, with $m_i = (1, 2, 2, \dots, 2, 1, 1)$;
3. singlet, $\underline{27}$, and $\overline{27}$ -plets of $E(6)$, corresponding to $m_i = (1, 2, 2, 3, 2, 1)$;
4. singlet of $E(8)$, with $m_i = (2, 3, 4, 6, 5, 4, 3, 2)$.

Therefore only these representations can be used to incorporate matter and Higgs fields in GUSTs with KMA on the level 1.

In principle it should be possible to construct explicitly an example of a level 1 KMA-representation of the simply laced \hat{g} algebra (A-, D-, E - types) from the level one representations of the Cartan subalgebra of g . This construction is achieved using the vertex operator of string, where these operators are assigned to a set of lattice point corresponding to the roots of a simply-laced Lie algebra g . In heterotic string approach the vertex operator for a gauge boson with momentum p and polarization ζ is a primary field of conformal dimension $(1/2, 1)$ and could be written in the form:

$$V^a = \zeta_\mu \psi_\mu(\bar{z}) J^a \exp(ipX), \quad p_\mu p^\mu = \zeta_\mu p^\mu = 0. \quad (32)$$

X_μ is the string coordinate and ψ^μ is a conformal dimension $(1/2, 0)$ Ramond-Neveu-Schwartz fermion.

2.1.2 The features of one level KMA in matter and Higgs representations in rank 8- and 16- GUST Constructions

For example, to describe chiral matter fermions in GUST with the gauge symmetry group $SU(5) \times U(1) \subset SO(10)$ the following sum of the level-one complex representations: $\underline{1}(-5/2) + \underline{\bar{5}}(+3/2) + \underline{10}(-1/2) = \underline{16}$ can be used. On the other side, as real representations of $SU(5) \times U(1) \subset SO(10)$, from which Higgs fields can arise, one can take for example $\underline{5} + \underline{\bar{5}}$ representations arising from real representation $\underline{10}$ of $SO(10)$. Also, real Higgs representations like $\underline{10}(-1/2) + \underline{\bar{10}}(+1/2)$ of $SU(5) \times U(1)$ originating from $\underline{16} + \underline{\bar{16}}$ of $SO(10)$, which has been used in ref. [10] for further symmetry breaking, are allowed.

Another example is provided by the decomposition of $SO(16)$ representations under $SU(8) \times U(1) \subset SO(16)$. Here only singlet, $v = \underline{16}$, $s = \underline{128}$ and $s' = \underline{128}'$ representations of $SO(16)$ are allowed by the KMA ($s = \underline{128}$ and $s' = \underline{128}'$ are the two nonequivalent, real spinor representations with the highest weights $\pi_{7,8} = 1/2(\epsilon_1 + \epsilon_2, + \dots + \epsilon_7 \mp \epsilon_8)$, $\epsilon_i \epsilon_j = \delta_{ij}$). From the item 2. we can obtain the following $SU(8) \times U(1)$ representations: singlet, $\underline{8} + \underline{\bar{8}}$ ($= \underline{16}$), $\underline{8} + \underline{56} + \underline{\bar{56}} + \underline{\bar{8}}$ ($= \underline{128}$) and $\underline{1} + \underline{28} + \underline{70} + \underline{+28} + \underline{\bar{1}}$ ($= \underline{128}'$). The highest weights of $SU(8)$ representations $\pi_1 = \underline{8}$, $\pi_7 = \underline{\bar{8}}$ and $\pi_3 = \underline{56}$, $\pi_5 = \underline{\bar{56}}$ are:

$$\begin{aligned} \pi_1 &= 1/8(7\epsilon_1 - \epsilon_2 - \epsilon_3 - \epsilon_4 - \epsilon_5 - \epsilon_6 - \epsilon_7 - \epsilon_8), \\ \pi_7 &= 1/8(\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 + \epsilon_5 + \epsilon_6 + \epsilon_7 - 7\epsilon_8), \\ \pi_3 &= 1/8(5\epsilon_1 + 5\epsilon_2 + 5\epsilon_3 - 3\epsilon_4 - 3\epsilon_5 - 3\epsilon_6 - 3\epsilon_7 - 3\epsilon_8), \\ \pi_5 &= 1/8(-3\epsilon_1 - 3\epsilon_2 - 3\epsilon_3 - 3\epsilon_4 - 3\epsilon_5 + 5\epsilon_6 + 5\epsilon_7 + 5\epsilon_8). \end{aligned} \quad (33)$$

Similarly, the highest weights of $SU(8)$ representations $\pi_2 = \underline{28}$, $\pi_6 = \underline{28}$ and $\pi_4 = \underline{70}$ are:

$$\begin{aligned}\pi_2 &= 1/4(3\epsilon_1 + 3\epsilon_2 - \epsilon_3 - \epsilon_4 - \epsilon_5 - \epsilon_6 - \epsilon_7 - \epsilon_8), \\ \pi_6 &= 1/4(\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 + \epsilon_5 + \epsilon_6 - 3\epsilon_7 - 3\epsilon_8), \\ \pi_4 &= 1/2(\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 - \epsilon_5 - \epsilon_6 - \epsilon_7 - \epsilon_8).\end{aligned}\tag{34}$$

However, as we will demonstrate, in each of the string sectors the generalized Gliozzi–Scherk–Olive projection (the GSO projection in particular guarantees modular invariance and supersymmetry of the theory and also give some non-trivial restrictions on gauge groups and its representations) necessarily eliminates either $\underline{128}$ or $\underline{128}'$. It is therefore important that, in order to incorporate chiral matter in the model, only one spinor representation is sufficient. Moreover, if one wants to solve the chirality problem applying further GSO projections (which break the gauge symmetry) also the representation $\underline{10}$ which otherwise, together with $\underline{10}$, could form real Higgs representation, disappears from this sector. Therefore, the existence of $\underline{10}_{-1/2} + \underline{10}_{1/2}$, needed for breaking $SU(5) \times U(1)$ is incompatible (by our opinion) with the possible solution of the chirality problem for the family matter fields.

Thus, in the rank eight group $SU(8) \times U(1) \subset SO(16)$ with Higgs representations from the level-one KMA only, one can not arrange for further symmetry breaking. Moreover, construction of the realistic fermion mass matrices seems to be impossible. In old-fashioned GUTs (see e.g.[23]), not originating from strings, the representations of level-two were commonly used to solve these problems.

The way out from this difficulty is based on the following important observations. Firstly, all higher-dimensional representations of (simple laced) groups like $SU(N)$, $SO(2N)$ or $E(6)$, which belong to the level-two of the KMA (according to the equation 29), appear in the direct product of the level- one representations:

$$R_G(x=2) \subset R_G(x=1) \times R_G'(x=1).\tag{35}$$

For example, the level-two representations of $SU(5)$:

$\underline{15}$, $\underline{24}$, $\underline{40}$, $\underline{45}$, $\underline{50}$, $\underline{75}$

will appear in the direct products of:

$\underline{5} \times \underline{5}$, $\underline{5} \times \underline{\bar{5}}$, $\underline{5} \times \underline{10}$, etc. respectively.

In the case of $SO(10)$ the level two representations:

$\underline{45}$, $\underline{54}$, $\underline{120}$, $\underline{126}$, $\underline{210}$, $\underline{144}$

can be obtained by the suitable direct products:

$\underline{10} \times \underline{10}$, $\underline{16} \times \underline{10}$, $\underline{16} \times \underline{16}$, $\underline{16} \times \underline{16}$.

The level-two representations:

$\underline{78}$, $\underline{351}$, $\underline{351}'$, $\underline{650}$

of $E(6)$ are factors of the decomposition of the direct products of:

$\underline{27} \times \underline{27}$ or $\underline{27} \times \underline{27}$.

The only exception from this rule is the $E(8)$ group, two level-two representations ($\underline{248}$ and $\underline{3875}$) of which cannot be constructed as a product of level-one representations [24].

Secondly, the diagonal (symmetric) subgroup G^{symm} of $G \times G$ effectively corresponds to the level-two KMA $g(x=1) \oplus g(x=1)$ [25, 26] because taking the $G \times G$ representations in the form (R_G, R'_G) of the $G \times G$, where R_G and R'_G belong to the level-one of G , one obtains representations of the form $R_G \times R'_G$ when one considers only the diagonal subgroup of $G \times G$. This observation is crucial, because such a construction allows one to obtain level-two representations. (This construction has implicitly been used in [26] (see also [25]) where we have constructed some examples of GUST with gauge symmetry realized as a diagonal subgroup of direct product of two rank eight groups $U(8) \times U(8) \subset SO(16) \times SO(16)$.)

In strings, however, not all level-two representations can be obtained in that way because, as we will demonstrate, some of them become massive (with masses of order of the Planck scale). The condition ensuring that in the string spectrum states transforming as a representation R are massless reads:

$$h(R) = \frac{Q_R}{2k + Q_{ADJ}} = \frac{Q_R}{2Q_M} \leq 1, \quad (36)$$

where Q_i is the quadratic Casimir invariant of the corresponding representations, and M has been already defined before (see eq. 31). Here the conformal weight is defined by $L_0|0\rangle = h(R)|0\rangle$,

$$L_0 = \frac{1}{2k + Q_\psi} \times \left(\sum_{a=1}^{a=\dim g} (T_0^a T_0^a + 2 \sum_{n=1}^{n=\inf} T_{-n}^a T_n^a) \right), \quad (37)$$

where $T_n^a|0\rangle = 0$ for $n > 0$. The condition (36), when combined with (29), gives a restriction at the rank of GUT's group ($r \leq 8$), whose representations can accomodate chiral matter fields. For example, for $G = SO(16)$ or $E(6) \times SU(3)$, representations $\underline{128}$, $(\underline{27}, \underline{3})$ ($h(\underline{128}) = 1$, $h(\underline{27}, \underline{3}) = 1$) respectively, satisfy both conditions. Obviously, these (important for incorporation of chiral matter) representations will exist at the level-two KMA of the symmetric subgroup of the group $G \times G$.

In general, condition (36) severely constrains massless string states transforming as $(R_G(x=1), R'_G(x=1))$ of the direct product $G \times G$. For example, for $SU(8) \times SU(8)$ and for $SU(5) \times SU(5)$ constructed from $SU(8) \times SU(8)$ only representations of the form

$$R_{N,N} = ((\underline{N}, \underline{N}) + h.c.), \quad ((\underline{N}, \bar{\underline{N}}) + h.c.); \quad (38)$$

with $h(R_{N,N}) = (N-1)/N$, where $N = 8$ or 5 respectively can be massless. For $SO(2N) \times SO(2N)$ massless states are contained only in representations

$$R_{v,v} = (\underline{2N}, \underline{2N}) \quad (39)$$

with $h(R_{v,v}) = 1$. Thus, for the GUSTs based on a diagonal subgroup $G^{symm} \subset G \times G$, G^{symm} - high dimensional representations, which are embedded in $R_G(x=1) \times R'_G(x=1)$ are also severely constrained by the condition (36).

For spontaneous breaking of $G \times G$ gauge symmetry down to G^{symm} (rank $G^{symm} = \text{rank } G$) one can use the direct product of representations $R_G(x=1) \times R_G(x=1)$, where $R_G(x=1)$ is the fundamental representation of $G = SU(N)$ or vector representation of $G = SO(2N)$. Furthermore, $G^{symm} \subset G \times G$ can subsequently be broken down to a smaller dimension gauge group (of the same rank as G^{symm}) through the VEVs of the adjoint representations which can appear as a result of $G \times G$ breaking. Alternatively, the real Higgs superfields (38) or (39) can directly break the $G \times G$ gauge symmetry down to a $G_1^{symm} \subset G^{symm}$ (rank $G_1^{symm} \leq \text{rank } G^{symm}$). For example when $G = SU(5) \times U(1)$ or $SO(10) \times U(1)$ $G \times G$ can directly be broken in that way down to $SU(3^c) \times G_{EW}^I \times G_{EW}^{II} \times \dots$

The above examples show clearly, that within the framework of GUSTs with the KMA one can get interesting gauge symmetry breaking chains including the realistic ones provided $G \times G$ gauge symmetry group is considered. However the lack of the higher dimensional representations (which are forbidden by 36) on the level-two KMA prevents the construction of the realistic fermion mass matrices. That is why we consider an extended grand unified string model of rank eight . $SO(16)$ or $E(6) \times SU(3)$ of $E(8)$.

The full chiral $SO(10) \times SU(3) \times U(1)$ matter multiplets can be constructed from $SU(8) \times U(1)$ -multiplets

$$(\underline{8} + \underline{56} + \bar{\underline{8}} + \bar{\underline{56}}) = \underline{128} \quad (40)$$

of $SO(16)$. In the 4-dimensional heterotic superstring with free complex world sheet fermions, in the spectrum of the Ramond sector there can appear also representations which are factors in the decomposition of $\underline{128}'$. In particular, $SU(5)$ -decouplets $(\underline{10} + \bar{\underline{10}})$ from $(\underline{28} + \bar{\underline{28}})$ of $SU(8)$. However their $U(1)_5$ hypercharge prevent using them for $SU(5) \times U(1)_5$ -symmetry breaking. Thus, in this approach we have only singlet and $(\underline{5} + \bar{\underline{5}})$ Higgs fields which can break the grand unified $SU(5) \times U(1)$ gauge symmetry. Therefore it is necessary (as we already explained) to construct rank eight GUST based on a diagonal subgroup $G^{symm} \subset G \times G$ primordial symmetry group, where in each rank eight group G the Higgs fields will appear only in singlets and in the fundamental representations as in (see 38).

A comment concerning $U(1)$ factors can be made here. Since the available $SU(5) \times U(1)$ decouplets have non-zero hypercharges with respect to $U(1)_5$ and $U(1)_H$, these $U(1)$ factors may remain unbroken down to the low energies in the model considered which seems to be very interesting.

2.2 GUST Constructions in Free Fermion Formulation.

2.2.1 Modular invariance and spin- basis.

A Sugawara- Sommerfeld construction of the Virasoro algebra in terms of bilinears in the Kac- Moody generators [21], [22] allows to get the following expression for the central Virasoro "charge":

$$c_g = \frac{2k \dim g}{2k + Q_\psi} = \frac{x \dim g}{x + \tilde{h}}. \quad (41)$$

In heterotic string theories [13, 14] $(N = 1 \text{ } SUSY)_{LEFT} (N = 0 \text{ } SUSY)_{RIGHT} \oplus \mathcal{M}_{c_L; c_R}$ with $d \leq 10$, the conformal anomalies of the space-time sector are canceled by the conformal anomalies of the internal sector $\mathcal{M}_{c_L; c_R}$, where $c_L = 15 - 3d/2$ and $c_R = 26 - d$ are the conformal anomalies in the left- and right-moving string sectors respectively.

In the fermionic formulation of the four-dimensional heterotic string theory in addition to the two transverse bosonic coordinates X_μ, \bar{X}_μ and their left-moving superpartners ψ_μ , the internal sector $\mathcal{M}_{c_L; c_R}$ contains 44 right-moving ($c_R = 22$) and 18 left-moving ($c_L = 9$) real fermions. The model is completely defined by a set Ξ of spin boundary conditions for all these world-sheet fermions. In a diagonal basis the vectors of Ξ are determined by the values of phases $\alpha(f) \in (-1, 1]$ fermions f acquire ($f \rightarrow -\exp(i\pi\alpha(f))f$) when parallel transported around the string. To construct the GUST according to the scheme outlined at the end of the previous section we consider three different bases each of them with six elements $B = b_1, b_2, b_3, b_4 \equiv S, b_5, b_6$. (See Tables 1, 4 and 7.)

Following [19] we construct the canonical basis in such a way that the vector $\bar{1}$, which belongs to Ξ , is the first element b_1 of the basis. The basis vector $b_4 = S$ is the generator of supersymmetry [20] responsible for the conservation of the space-time $SUSY$.

We have chosen a basis in which all left movers ($\psi_\mu; \chi_i, y_i, \omega_i; i = 1, \dots, 6$) (on which the world sheet supersymmetry is realized nonlinearly) as well as 12 right movers ($\bar{\varphi}_k; k = 1, \dots, 12$) are real whereas $(8 + 8)$ right movers $\bar{\Psi}_A, \bar{\Phi}_M$ are complex. Such a construction corresponds to $SU(2)^6$ group of automorphisms of the left supersymmetric sector of a string. Right- and left-moving real fermions can be used for breaking G^{comp} symmetry [20]. In order to have a possibility to reduce the rank of the compactified group G^{comp} , we have to select the spin boundary conditions for the maximal possible number, $N_{LR} = 12$, of left-moving, $\chi_{3,4,5,6}, y_{1,2,5,6}, \omega_{1,2,3,4}$, and right-moving, $\bar{\phi}^{1, \dots, 12}$ ($\bar{\phi}^p = \bar{\varphi}_p, p = 1, \dots, 12$) real fermions. The KMA based on 16 complex right moving fermions gives rise to the "observable" gauge group, G^{obs} , with:

$$rank(G^{obs}) \leq 16. \quad (42)$$

The study of the Hilbert spaces of the string theories is connected to the problem of finding all possible choices of the GSO coefficients $\mathcal{C} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, such that the one-loop partition function

$$Z = \sum_{\alpha, \beta} \mathcal{C} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \prod_f Z \begin{bmatrix} \alpha_f \\ \beta_f \end{bmatrix} \quad (43)$$

and its multiloop counterparts are all modular invariant. In this formula $\mathcal{C} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ are GSO coefficients, α and β are $(k+l)$ -component spin-vectors $\alpha = [\alpha(f_1^r), \dots, \alpha(f_k^r); \alpha(f_1^c), \dots, \alpha(f_l^c)]$, the components α_f, β_f specify the spin structure of the f th fermion and $Z[\dots]$ – corresponding one-fermion partition functions on torus: $Z[\dots] = \text{Tr} \exp[2\pi i H_{(sect.)}]$.

The physical states in the Hilbert space of a given sector α are obtained acting on the vacuum $|0\rangle_\alpha$ with the bosonic and fermionic operators with frequencies

$$n(f) = 1/2 + 1/2\alpha(f), \quad n(f^*) = 1/2 - 1/2\alpha(f^*) \quad (44)$$

and subsequently applying the generalized GSO projections. The physical states satisfy the Virasoro condition:

$$M_L^2 = -1/2 + 1/8 (\alpha_L \cdot \alpha_L) + N_L = -1 + 1/8 (\alpha_R \cdot \alpha_R) + N_R = M_R^2, \quad (45)$$

where $\alpha = (\alpha_L, \alpha_R)$ is a sector in the set Ξ , $N_L = \sum_L(\text{frequencies})$ and $N_R = \sum_R(\text{freq.})$.

We keep the same sign convention for the fermion number operator F as in [20]. For complex fermions we have $F_\alpha(f) = 1$, $F_\alpha(f^*) = -1$ with the exception of the periodic fermions for which we get $F_{\alpha=1}(f) = -1/2(1 - \gamma_{5f})$, where $\gamma_{5f}|\Omega\rangle = |\Omega\rangle$, $\gamma_{5f}b_o^+|\Omega\rangle = -b_o^+|\Omega\rangle$.

The full Hilbert space of the string theory is constructed as a direct sum of different sectors $\sum_i m_i b_i$, ($m_i = 0, 1, \dots, N_i$), where the integers N_i define additive groups $Z(b_i)$ of the basis vectors b_i . The generalized GSO projection leaves in sectors α those states, whose b_i -fermion number satisfies:

$$\exp(i\pi b_i F_\alpha) = \delta_\alpha \mathcal{C}^* \begin{bmatrix} \alpha \\ b_i \end{bmatrix}, \quad (46)$$

where the space-time phase $\delta_\alpha = \exp(i\pi\alpha(\psi_\mu))$ is equal -1 for the Ramond sector and $+1$ for the Neveu-Schwarz sector.

2.2.2 $SU(5) \times U(1) \times SU(3) \times U(1)$ - Model 1.

Model 1 is defined by 6 basis vectors given in Table 1 which generates the $Z_2 \times Z_4 \times Z_2 \times Z_2 \times Z_8 \times Z_2$ group under addition.

In our approach the basis vector b_2 is constructed as a complex vector with the $1/2$ spin-boundary conditions for the right-moving fermions Ψ_A , $A = 1, \dots, 8$. Initially it generates chiral matter fields in the $\underline{8} + \underline{56} + \underline{56} + \underline{8}$ representations of $SU(8) \times U(1)$, which subsequently are decomposed under $SU(5) \times U(1) \times SU(3) \times U(1)$ to which $SU(8) \times U(1)$ gets broken by applying the b_5 GSO projection.

Generalized GSO projection coefficients are originally defined up to fifteen signs some of which, are fixed by the supersymmetry conditions. Below, in Table 1, we present a set of numbers

$$\gamma \begin{bmatrix} b_i \\ b_j \end{bmatrix} = \frac{1}{i\pi} \log \mathcal{C} \begin{bmatrix} b_i \\ b_j \end{bmatrix}.$$

which we use as basis for our GSO projections.

In our case of the $Z_2^4 \times Z_4 \times Z_8$ model, we initially have 256×2 sectors. After applying the GSO-projections we get only 49×2 sectors containing massless states, which depending on the vacuum energy values, E_L^{vac} and E_R^{vac} , can be naturally divided into some classes and which determine the GUST representations.

Generally RNS (Ramond – Neveu-Schwarz) sector (built on vectors b_1 and $S = b_4$) has high symmetry including $N = 4$ supergravity and gauge $SO(44)$ symmetry. Corresponding gauge bosons are constructed as follows:

$$\begin{aligned} \psi_{1/2}^\mu |0\rangle_L \otimes \Psi_{1/2}^I \Psi_{1/2}^J |0\rangle_R, \\ \psi_{1/2}^\mu |0\rangle_L \otimes \Psi_{1/2}^I \Psi_{1/2}^{*J} |0\rangle_R, \quad I, J = 1, \dots, 22 \end{aligned} \quad (47)$$

While $U(1)_J$ charges for Cartan subgroups is given by formula $Y = \frac{\alpha}{2} + f$ (where F — fermion number, see (46)), it is obvious that states (47) generate root lattice for $SO(44)$:

$$\pm \varepsilon_I \pm \varepsilon_J \quad (I \neq J); \quad \pm \varepsilon_I \mp \varepsilon_J \quad (48)$$

The others vectors breaks $N = 4$ SUSY to $N = 1$ and gauge group $SO(44)$ to $SO(2)_{1,2,3}^3 \times SO(6)_4 \times [SU(5) \times U(1) \times SU(3)_H \times U(1)_H]^2$, see Figure 2.

Generally, additional basis vectors can generate extra vector bosons and extend gauge group that remains after applying GSO-projection to RNS-sector. In our case dangerous sectors are: $2b_2 + nb_5$, $n = 0, 2, 4, 6$; $2b_5$; $6b_5$. But our choice of GSO coefficients cancels all the vector states in these sectors. Thus gauge bosons in this model appear only from RNS-sector.

In NS sector the b_3 GSO projection leaves $(5, \bar{3}) + (\bar{5}, 3)$ Higgs superfields:

$$\chi_{1/2}^{1,2} |\Omega >_L \otimes \Psi_{1/2}^a \Psi_{1/2}^{i*}; \quad \Psi_{1/2}^{a*} \Psi_{1/2}^i |\Omega >_R \quad \text{and exchange } \Psi \longrightarrow \Phi, \quad (49)$$

where $a, b = 1, \dots, 5$, $i, j = 1, 2, 3$.

Four $(3_H + 1_H)$ generations of chiral matter fields from $(SU(5) \times SU(3))_I$ group forming $SO(10)$ -multiplets $(\underline{1}, \underline{3}) + (\underline{\bar{5}}, \underline{3}) + (\underline{10}, \underline{3})$; $(\underline{1}, \underline{1}) + (\underline{\bar{5}}, \underline{1}) + (\underline{10}, \underline{1})$ are contained in b_2 and $3b_2$ sectors. Applying b_3 GSO projection to the $3b_2$ sector yields the following massless states:

$$\begin{aligned} b_{\psi_{12}}^+ b_{\chi_{34}}^+ b_{\chi_{56}}^+ |\Omega >_L \otimes & \left\{ \Psi_{3/4}^{i*}, \Psi_{1/4}^a \Psi_{1/4}^b \Psi_{1/4}^c, \Psi_{1/4}^a \Psi_{1/4}^i \Psi_{1/4}^j \right\} |\Omega >_R, \\ b_{\chi_{12}}^+ b_{\chi_{34}}^+ b_{\chi_{56}}^+ |\Omega >_L \otimes & \left\{ \Psi_{3/4}^{a*}, \Psi_{1/4}^a \Psi_{1/4}^b \Psi_{1/4}^i, \Psi_{1/4}^i \Psi_{1/4}^j \Psi_{1/4}^k \right\} |\Omega >_R \end{aligned} \quad (50)$$

with the space-time chirality $\gamma_{5\psi_{12}} = -1$ and $\gamma_{5\psi_{12}} = 1$, respectively. In these formulae the Ramond creation operators $b_{\psi_{1,2}}^+$ and $b_{\chi_{\alpha,\beta}}^+$ of the zero modes are built of a pair of real fermions (as indicated by double indices): $\chi_{\alpha,\beta}$, $(\alpha, \beta) = (1, 2), (3, 4), (5, 6)$. Here, as in (49) indices take values $a, b = 1, \dots, 5$ and $i, j = 1, 2, 3$ respectively.

We stress that without using the b_3 projection we would get matter supermultiplets belonging to real representations only i.e. "mirror" particles would remain in the spectrum. The b_6 projection instead, eliminates all chiral matter superfields from $U(8)^{II}$ group.

Since the matter fields form the chiral multiplets of $SO(10)$, it is possible to write down $U(1)_{Y_5}$ -hypercharges of massless states. In order to construct the right electromagnetic charges for matter fields we must define the hypercharges operators for the observable $U(8)^I$ group as

$$Y_5 = \int_0^\pi d\sigma \sum_a \Psi^{*a} \Psi^a, \quad Y_3 = \int_0^\pi d\sigma \sum_i \Psi^{*i} \Psi^i \quad (51)$$

and analogously for the $U(8)^{II}$ group.

Then the orthogonal combinations

$$\tilde{Y}_5 = \frac{1}{4}(Y_5 + 5Y_3), \quad \tilde{Y}_3 = \frac{1}{4}(Y_3 - 3Y_5), \quad (52)$$

play the role of the hypercharge operators of $U(1)_{Y_5}$ and $U(1)_{Y_H}$ groups, respectively. In a Table 3 we give the hypercharges $\tilde{Y}_5^I, \tilde{Y}_3^I, \tilde{Y}_5^{II}, \tilde{Y}_3^{II}$.

The full list of states in this model is given in a Table 3. For fermion states only sectors with positive (left) chirality is written. Superpartners arises from sectors with $S = b_4$ -component changed by 1. Chirality under hidden $SO(2)_{1,2,3}^3 \times SO(6)_4$ is defined as $\pm_1, \pm_2, \pm_3, \pm_4$ respectively. Low signs in item 5 and 6 correspond to sectors with components given in brackets.

In the next section we discuss the problem of rank eight GUST gauge symmetry breaking. The matter is that according to the results of section 2.1 the Higgs fields ($\underline{10}_{1/2} + \bar{\underline{10}}_{-1/2}$) do not appear.

2.2.3 $SU(5) \times U(1) \times SU(3) \times U(1)$ Model 2.

Consider then another $[U(5) \times U(3)]^2$ model which after breaking gauge symmetry by Higgs mechanism leads to the spectrum similar to Model 1.

This model is defined by basis vectors given in a Table 4 with the $Z_2^4 \times Z_6 \times Z_{12}$ group under addition.

Table 4: Basis of the boundary conditions for Model 2.

Vectors	$\psi_{1,2}$	$\chi_{1,\dots,6}$	$y_{1,\dots,6}$	$\omega_{1,\dots,6}$	$\bar{\varphi}_{1,\dots,12}$	$\Psi_{1,\dots,8}$	$\Phi_{1,\dots,8}$
b_1	11	1^6	1^6	1^6	1^{12}	1^8	1^8
b_2	11	1^6	0^6	0^6	0^{12}	$1^5 1/3^3$	0^8
b_3	11	$1^2 0^2 1^2$	0^6	$0^2 1^2 0^2$	$0^8 1^4$	$1/2^5 1/6^3$	$-1/2^5 1/6^3$
$b_4 = S$	11	$1^2 0^4$	$0^2 1^2 0^2$	$0^4 1^2$	0^{12}	0^8	0^8
b_5	11	$1^4 0^2$	$0^4 1^2$	0^6	$1^8 0^4$	$1^5 0^3$	$0^5 1^3$
b_6	11	$0^2 1^2 0^2$	$1^2 0^4$	$0^4 1^2$	$1^2 0^2 1^6 0^2$	1^8	0^8

GSO coefficients are given in Table 5.

Table 5: The choice of the GSO basis $\gamma[b_i, b_j]$. Model 2. (i numbers rows and j – columns)

	b_1	b_2	b_3	b_4	b_5	b_6
b_1	0	1	$1/2$	0	0	0
b_2	0	$2/3$	$-1/6$	1	0	1
b_3	0	$1/3$	$5/6$	1	0	0
b_4	0	0	0	0	0	0
b_5	0	1	$-1/2$	1	1	1
b_6	0	1	$1/2$	1	0	1

The given model corresponds to the following chain of the gauge symmetry breaking:

$$E_8^2 \longrightarrow SO(16)^2 \longrightarrow U(8)^2 \longrightarrow [U(5) \times U(3)]^2 .$$

When the breaking of $U(8)^2$ -group to $[U(5) \times U(3)]^2$ determined by basis vector b_5 , and $N=2$ SUSY \longrightarrow $N=1$ SUSY determined by basis vector b_6 .

It is interesting to note how the $U(8)^2$ gauge group restored by sectors $4b_3$, $8b_3$, $2b_2 + c.c.$ and $4b_2 + c.c.$

The full massless spectrum for given model is given in Table 6. By analogy with Table 3 only fermion states with positive chirality is written and obviously vector supermultiplets are absent. Hypercharges determines by formula:

$$Y_n = \sum_{k=1}^n (\alpha_k/2 + F_k) .$$

The given model possesses by the hidden gauge symmetry $SO(16)_1 \times SO(2)_{2,3,4}^3$. The corresponding chirality is given in column SO_{hid} . The sectors are divided by horizontal lines and without including the b_5 -vector form $SU(8)$ -multiplets.

For example, let us consider row No 2. In sectors b_2 , $5b_2$ in addition to states $(1, \bar{3})$ and $(5, \bar{3})$ the $(10, 3)$ -state appears, and in the sector $3b_2$ besides the $(\bar{10}, 1)$ - the states $(1, 1)$ and $(\bar{5}, 1)$ survive too. All these states form $\bar{8} + 56$ representation of the $SU(8)^I$ group.

Analogically we can get the full structure of the theory according $U(8)^I \times U(8)^{II}$ -group. (For correct restoration of the $SU(8)^{II}$ -group we must invert 3 and $\bar{3}$ representations.)

In Model 2 matter fields appear both in $U(8)^I$ and $U(8)^{II}$ groups. This is the main difference with comparing of the Model 1. However, note that in the Model 2 similarly to the Model 1 all gauge fields appear in RNS-sector only and $10 + \bar{10}$ representation (which can be the Higgs field for gauge symmetry breaking) is absent.

Table 6: The list of quantum numbers of the states. Model 2.

N ^o	$b_1, b_2, b_3, b_4, b_5, b_6$	SO_{hid}	$U(5)^I$	$U(3)^I$	$U(5)^{II}$	$U(3)^{II}$	Y_5^I	Y_3^I	Y_5^{II}	Y_3^{II}
1	RNS 0 0 4 1 0 0 0 0 8 1 0 0	$6_1 \ 2_2$ $2_3 \ 2_4$	1	1	1	1	0	0	0	0
			1	1	1	1	0	0	0	0
			5	1	$\bar{5}$	1	1	0	-1	0
			1	3	1	3	0	-1	0	-1
			1	$\bar{3}$	1	$\bar{3}$	0	1	0	1
2	0 1 0 0 0 0 0 3 0 0 0 0		5	3	1	1	-3/2	-1/2	0	0
			1	$\bar{3}$	1	1	5/2	-1/2	0	0
			$\bar{10}$	1	1	1	1/2	3/2	0	0
3	0 1 1 0 0 0 0 0 3 6 0 0 0		1	1	$\bar{10}$	3	0	0	1/2	1/2
			1	1	5	1	0	0	-3/2	-3/2
			1	1	1	1	0	0	5/2	-3/2
4	0 2 3 0 0 0	$-3 \ \pm_4$	1	3	1	1	-5/4	-1/4	5/4	3/4
5	0 0 3 0 0 0	$+3 \ \pm_4$	1	1	$\bar{5}$	1	-5/4	3/4	1/4	3/4
6	0 0 9 0 0 0	$+3 \ \pm_4$	1	1	5	1	5/4	-3/4	-1/4	-3/4
7	0 4 9 0 0 0	$-3 \ \pm_4$	1	3	1	1	5/4	1/4	-5/4	-3/4
8,9	0 5 0 1 0 1 0 3 0 1 0 1	$-1 \ \pm_3$	1	3	1	1	0	-1	0	0
		$+1 \ +_3$	5	1	1	1	1	0	0	0
		$+1 \ -_3$	$\bar{5}$	1	1	1	-1	0	0	0
		$-1 \ +_3$	1	1	5	1	0	0	1	0
		$-1 \ -_3$	1	1	$\bar{5}$	1	0	0	-1	0
	0 5 8 1 0 1	$+1 \ +_3$	1	1	1	$\bar{3}$	0	0	0	1
10	0 3 3 0 0 1	$+1 \ \pm_4$	1	1	1	1	-5/4	3/4	5/4	3/4
11	1 0 3 0 0 1	$\pm_2 \ -_3$	1	1	5	1	-1/4	3/4	-5/4	-3/4
	1 2 1 1 0 0 1	$\pm_2 \ -_3$	1	1	1	$\bar{3}$	-5/4	3/4	-5/4	1/4
12	1 0 9 0 0 1	$\pm_2 \ +_3$	$\bar{5}$	1	1	1	1/4	-3/4	5/4	3/4
	1 4 9 0 0 1	$\pm_2 \ +_3$	1	$\bar{3}$	1	1	5/4	1/4	5/4	3/4
13	0 0 0 1 1 1	$\pm_2 \ +_3$	1	1	1	1	0	-3/2	0	3/2
	0 2 0 1 1 1	$\pm_2 \ -_3$	1	3	1	1	0	1/2	0	3/2
	0 2 8 1 1 1	$\pm_2 \ -_3$	1	1	1	$\bar{3}$	0	-3/2	0	-1/2
	0 4 8 1 1 1	$\pm_2 \ +_3$	1	3	1	$\bar{3}$	0	1/2	0	-1/2
	1 0 3 1 1 1	$+1 \ +_3$	1	1	1	1	5/4	3/4	-5/4	3/4
	1 0 9 1 1 1	$+1 \ +_3$	1	1	1	1	-5/4	-3/4	5/4	-3/4
	1 3 3 0 1 1	$-1 \ -_3$	1	1	1	1	-5/4	-3/4	-5/4	3/4
	1 3 9 0 1 1	$-1 \ +_3$	1	1	1	1	5/4	3/4	5/4	-3/4

2.2.4 $SO(10) \times SU(3) \times U(1)$ Model 3.

As an illustration we can consider the GUST construction involving $SO(10)$ as GUT gauge group. We consider the set consists of six vectors $B = b_1, b_2, b_3, b_4 \equiv S, b_5, b_6$ given in Table 7.

Table 7: Basis of the boundary conditions for the Model 3.

Vectors	$\psi_{1,2}$	$\chi_{1,...,6}$	$y_{1,...,6}$	$\omega_{1,...,6}$	$\bar{\varphi}_{1,...,12}$	$\Psi_{1,...,8}$	$\Phi_{1,...,8}$
b_1	11	111111	111111	111111	1^{12}	1^8	1^8
b_2	11	111111	000000	000000	0^{12}	$1^5 1/3^3$	0^8
b_3	11	000000	111111	000000	$0^8 1^4$	$0^5 1^3$	$0^5 1^3$
$b_4 = S$	11	110000	001100	000011	0^{12}	0^8	0^8
b_5	11	111111	000000	000000	0^{12}	0^8	$1^5 1/3^3$
b_6	11	001100	110000	000011	$1^2 0^2 1^6 0^2$	1^8	0^8

GSO projections are given in Table 8. It is interesting to note that in this model the horizontal gauge symmetry $U(3)$ extends to $SU(4)$. Vector bosons which are needed for this appear in sectors $2b_2$ ($4b_2$) and $2b_5$ ($4b_5$). For further breaking $SU(4)$ to $SU(3) \times U(1)$ we need an additional basis spin-vector.

So, the given model possesses gauge group $G^{comp.} \times [SO(10) \times SU(4)]^2$ and matter fields appear both in first and in second group symmetrically. Sectors $3b_2$ and $5b_2 + c.c.$ give the matter fields $(\underline{16}, \underline{4}; \underline{1}, \underline{1})$ (first group) and sectors $3b_5$ and $5b_5 + c.c.$ give the matter fields $(\underline{1}, \underline{1}; \underline{16}, \underline{4})$ (second group).

Of course for getting a realistic model we must add some basis vectors which give addition GSO-projections.

Table 8: The choice of the GSO basis $\gamma[b_i, b_j]$. Model 3. (i numbers rows and j – columns)

	b_1	b_2	b_3	b_4	b_5	b_6
b_1	0	1	0	0	1	0
b_2	0	$2/3$	1	1	1	1
b_3	0	1	0	1	1	1
b_4	0	0	0	0	0	0
b_5	0	1	1	1	$2/3$	0
b_6	0	1	0	1	1	1

The condition of generation chirality in this model results in choice of Higgs fields as a vector representations of $SO(10)$ ($\underline{16} + \bar{\underline{16}}$ are absent). According to conclusion (39) the only Higgs fields $(\underline{10}, \underline{1}; \underline{10}, \underline{1})$ of $(SO(10) \times SU(4))^{\times 2}$ appear in model (from RNS-sector) which can be used for GUT gauge symmetry.

2.2.5 $E_6 \times SU(3)$ tree generations model (Model 4).

This model illustrates a branch of E_8 breaking $E_8 \rightarrow E_6 \times SU(3)$ and is an interesting result on a way to obtain three generations with gauge horizontal symmetry. Basis of the boundary conditions (see Table 9) is rather simple but there are some subtle points. In [35] the possible left parts of basis vectors were worked out, see it for details. We just use the notation given in [35] (hat on left part means complex fermion, other fermions on the

left sector are real, all of the right movers are complex) and an example of commuting set of vectors.

Table 9: Basis of the boundary conditions for the Model 4.

Vectors	$\psi_{1,2}$	$\chi_{1,...,9}$	$\omega_{1,...,9}$	$\bar{\varphi}_{1,...,6}$	$\Psi_{1,...,8}$	$\Phi_{1,...,8}$
b_1	11	1^9	1^9	1^6	1^8	1^8
b_2	11	$\frac{1}{3}, 1; -\frac{2}{3}, 0, 0, \frac{1}{3}$	$\frac{1}{3}, 1; -\frac{2}{3}, 0, 0, \frac{1}{3}$	$\frac{2}{3}^3 - \frac{2}{3}^3$	$0^2 - \frac{2}{3}^6$	$1^2 \frac{1}{3}^6$
b_3	00	0^9	0^9	0^6	1^8	0^8
b_4	11	$\hat{1}, 1; \hat{0}, 0, 0, \hat{0}$	$\hat{1}, 1; \hat{0}, 0, 0, \hat{0}$	0^6	0^8	0^8

A construction of an $E_6 \times SU(3)$ group caused us to use rational for left boundary conditions. It seems that it is the only way to obtain such a gauge group with appropriate matter contents.

The model has $N = 2$ SUSY. We can also construct model with $N = 0$ but according to [35] using vectors that can give rise to $E_6 \times SU(3)$ (with realistic matter fields) one cannot obtain $N = 1$ SUSY.

Table 10: The choice of the GSO basis $\gamma[b_i, b_j]$. Model 4. (i numbers rows and j – columns).

	b_1	b_2	b_3	b_4
b_1	0	1/3	1	1
b_2	1	1	1	1
b_3	1	1	1	0
b_4	1	1/3	1	1

Let us give a brief review of the model contents. First notice that all superpartners of states in sector α are found in sector $\alpha + b_4$ as in all previous models. Although the same sector may contain, say, matter fields and gauginos simultaneously.

The observable gauge group $(SU(3)_H^I \times E_6^I) \times (SU(3)_H^{II} \times E_6^{II})$ and hidden group $SU(6) \times U(1)$ are rising up from sectors NS, b_3 and $3b_2 + b_4$. Matter fields in representations $(\mathbf{3}, \mathbf{27}) + (\bar{\mathbf{3}}, \mathbf{27})$ for each $SU(3)_H \times E_6$ group are found in sectors $3b_2$, $b_3 + b_4$ and b_4 . Also there are some interesting states in sectors b_2 , $b_2 + b_3$, $2b_2 + b_3 + b_4$, $2b_2 + b_4$ and $5b_2$, $5b_2 + b_3$, $4b_2 + b_3 + b_4$, $4b_2 + b_4$ that form representations $(\bar{\mathbf{3}}, \mathbf{3})$ and $(\mathbf{3}, \bar{\mathbf{3}})$ of the $SU(3)_H^I \times SU(3)_H^{II}$ group. This states are singlets under both E_6 groups.

We suppose that the model permits further breaking of E_6 down to other grand unification groups, but problem with breaking supersymmetry $N = 2 \rightarrow N = 1$ is a great obstacle on this way.

2.3 Gauge Symmetry Breaking and GUST Spectrum

Let us consider the Model 1 in details. In the Model 1 there exists a possibility to break the GUST group $(U(5) \times U(3))^I \times (U(5) \times U(3))^{II}$ down to the symmetric group by the ordinary Higgs mechanism [13]:

$$G^I \times G^{II} \rightarrow G^{symm} \rightarrow \dots \quad (53)$$

To achieve such breaking one can use nonzero vacuum expectation values of the tensor Higgs fields (see Table 3, row No 1), contained in the $2b_2 + 2(6)b_5(+S)$ sectors which transform under the $(SU(5) \times U(1) \times SU(3) \times U(1))^{symm}$ group in the following way:

$$\begin{aligned} (\underline{5}, \underline{1}; \underline{5}, \underline{1})_{(-1,0;-1,0)} &\rightarrow (\underline{24}, \underline{1})_{(0,0)} + (\underline{1}, \underline{1})_{(0,0)}; \\ (\underline{1}, \underline{3}; \underline{1}, \underline{3})_{(0,1;0,1)} &\rightarrow (\underline{1}, \underline{8})_{(0,0)} + (\underline{1}, \underline{1})_{(0,0)}, \end{aligned} \quad (54)$$

$$\begin{aligned} (\underline{5}, \underline{1}; \underline{1}, \underline{3})_{(-1,0;0,1)} &\rightarrow (\underline{\bar{5}}, \underline{3})_{(1,1)}; \\ (\underline{1}, \underline{3}; \underline{5}, \underline{1})_{(0,1;-1,0)} &\rightarrow (\underline{5}, \underline{3})_{(-1,-1)}. \end{aligned} \quad (55)$$

The diagonal vacuum expectation values for the Higgs fields (54) break the the GUST group $(U(5) \times U(3))^I \times (U(5) \times U(3))^{II}$ down to the "skew"-symmetric group with the generators Δ_{symm} of the form:

$$\Delta_{symm}(t) = -t^* \times 1 + 1 \times t, \quad (56)$$

The corresponding hypercharge of the symmetric group reads:

$$\bar{Y} = \tilde{Y}^{II} - \tilde{Y}^I. \quad (57)$$

Similarly, for the electromagnetic charge we get:

$$\begin{aligned} Q_{em} &= Q^{II} - Q^I = \\ &= (T_5^{II} - T_5^I) + \frac{2}{5}(\tilde{Y}_5^{II} - \tilde{Y}_5^I) = \bar{T}_5 + \frac{2}{5}\bar{Y}_5, \end{aligned} \quad (58)$$

where $T_5 = diag(\frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{2}{5}, -\frac{3}{5})$. Note, that this charge quantization does not lead to exotic states with fractional electromagnetic charges

(e.g. $Q_{em} = \pm 1/2, \pm 1/6$).

Thus, in the breaking scheme (56) it is possible to avoid colour singlet states with fractional electromagnetic charges, to achieve desired GUT breaking and moreover to get the usual value for the weak mixing angle at the unification scale (see (18)).

Adjoint representations which appear on the *rhs* of (54) can be used for further breaking of the symmetric group. This can lead to the final physical symmetry

$$(SU(3^c) \times SU(2_{EW}) \times U(1_Y) \times U(1)') \times (SU(3_H) \times U(1_H)) \quad (59)$$

with low-energy gauge symmetry of the quark – lepton generations with an additional $U(1)'$ -factor.

Note, that using the same Higgs fields as in (54), there exists also another, interesting way of breaking the $G^I \times G^{II}$ gauge symmetry:

$$\begin{aligned} G^I \times G^{II} &\rightarrow SU(3^c) \times SU(2)_{EW}^I \times SU(2)_{EW}^{II} \times U(1_{\bar{Y}}) \times \\ &\times SU(3_H)^I \times SU(3_H)^{II} \times U(1_{\bar{Y}_H}) \rightarrow \end{aligned} \quad (60)$$

It is attractive because it naturally solves the Higgs doublet–triplet mass splitting problem with rather low energy scale of GUST symmetry breaking [34].

In turn, the Higgs fields $\hat{h}_{(\Gamma,N)}$ from the NS sector

$$(\underline{5}, \underline{3})_{(-1,-1)} + (\underline{5}, \underline{3})_{(1,1)} \quad (61)$$

originates from N=2 SUSY vector representation 63 of $SU(8)^I$ (or $SU(8)^{II}$) by applying the b_5 GSO projection (see Fig. 2). These Higgs fields (and fields (55)) can be used for constructing chiral fermion (see Table 3, row No 2) mass matrices.

The b spin boundary conditions (Tabl.1) generate chiral matter and Higgs fields with the $GUST$ gauge symmetry $G_{comp} \times (G^I \times G^{II})_{obs}$ (where $G_{comp} = U(1)^3 \times SO(6)$ and $G^{I,II}$ have been already defined). The chiral matter spectrum, which we denote $\hat{\Psi}_{(\Gamma,N)}$ with $(\Gamma = \underline{1}, \underline{5}, \underline{10}; N = \underline{3}, \underline{1})$, consists of $N_g = 3_H + 1_H$ families. See Table 3, row No 2 for the $((SU(5) \times U(1)) \times (SU(3) \times U(1))_H)^{symm}$ quantum numbers.

The $SU(3_H)$ anomalies of the matter fields (row No 2) are naturally canceled by the chiral "horizontal" superfields forming two sets: $\hat{\Psi}_{(1,N;1,N)}^H$ and $\hat{\Phi}_{(1,N;1,N)}^H$, $\Gamma = \underline{1}$, $N = \underline{1}, \underline{3}$, (with both $SO(2)_2$ chiralities, see Table 3, row No 3, 4).

The horizontal fields (No 3, 4) compensate all $SU(3)^I$ anomalies introduced by the chiral matter spectrum (No 2) of the $(U(5) \times U(3))^I$ group (due to b_6 GSO projection the chiral fields of the $(U(5) \times U(3))^{II}$ group disappear from the final string spectrum). Performing the decomposition of fields (No 3, 4) under $(SU(5) \times SU(3))^{symm}$ we get (among other) three "horizontal" fields:

$$(\underline{1}, \underline{3})_{(0,-1)}, (\underline{1}, \underline{1})_{(0,-3)}, (\underline{1}, \underline{6})_{(0,1)}, \quad (62)$$

coming from $\hat{\Psi}_{(\underline{1},\underline{3};\underline{1},\underline{1})}^H$, (or $\hat{\Psi}_{(\underline{1},\underline{1};\underline{1},\underline{3})}^H$), $\hat{\Psi}_{(\underline{1},\underline{1};\underline{1},\underline{1})}^H$ and $\hat{\Psi}_{(\underline{1},\underline{3};\underline{1},\underline{3})}^H$ respectively which make the low energy spectrum of the resulting model (60) $SU(3_H)^{symm}$ - anomaly free. The other fields arising from (rows No 3, 4, Table 3) form anomaly-free representations of $(SU(3_H) \times U(1_H))^{symm}$.

$$2(\underline{1}, \underline{1})_{(0,0)}, (\underline{1}, \underline{3})_{(0,-1(2))} + (\underline{1}, \underline{3})_{(0,1(-2))}, (\underline{1}, \underline{8})_{(0,0)}. \quad (63)$$

The superfields $\hat{\phi}_{(\Gamma,N)} + h.c.$, where $(\Gamma = \underline{1}, \underline{5}; N = \underline{1}, \underline{3})$, from the Table 3, row No 5 forming representations of $(U(5) \times U(3))^{I,II}$ have either Q^I or Q^{II} exotic fractional charges. Because of the strong G^{comp} gauge forces these fields may develop the double scalar condensate $\langle \hat{\phi}\hat{\phi} \rangle$, which can also serve for $U(5) \times U(5)$ gauge symmetry breaking. For example, the composite condensate $\langle \hat{\phi}_{(5,1;1,1)}\hat{\phi}_{(1,1;5,1)} \rangle$ can break the $U(5) \times U(5)$ gauge symmetry down to the symmetric diagonal subgroup with generators of the form

$$\Delta_{symm}(t) = t \times 1 + 1 \times t, \quad (64)$$

so for the electromagnetic charges we would have the form

$$Q_{em} = Q^{II} + Q^I. \quad (65)$$

leading again to no exotic, fractionally charged states in the low-energy string spectrum.

The superfields which transform nontrivially under the compactified group $G^{comp} = SO(6) \times SO(2)^{\times 3}$, (denoted as $\hat{\sigma} + h.c.$), and which are singlets of $(SU(5) \times SU(3)) \times (SU(5) \times SU(3))$, arise in three sectors, see Table 3, row No 6. The superfields $\hat{\sigma}$ form the spinor representations $\underline{4} + \bar{\underline{4}}$ of $SO(6)$ and they are also spinors of one of the $SO(2)$ groups. They have following hypercharges $\tilde{Y}_5^{I,II}$, $\tilde{Y}_3^{I,II}$:

$$\tilde{Y} = (5/4, \mp 3/4; 5/4, \mp 3/4), \tilde{Y} = (5/4, 3/4; -5/4, -3/4). \quad (66)$$

With respect to the diagonal G^{symm} group with generators given by (56) or (64), the fields $\hat{\sigma}$ from sets a), b) or the set c), are of zero hypercharges and can, therefore, be used for breaking the $SO(6) \times SO(2)^{\times 3}$ group.

Note, that for the fields $\hat{\phi}$ and for the fields $\hat{\sigma}$ any other electromagnetic charge quantization different than (58) or (65) would lead to "quarks" and "leptons" with the exotic fractional charges, for example, for the $\underline{5}$ - and $\underline{1}$ - multiplets according to the values of hypercharges (see eqs.66) the generator Q^{II} (or Q^I) has the eigenvalues $(\pm 1/6, \pm 1/6, \pm 1/6, \pm 1/2, \mp 1/2)$ or $\pm 1/2$, respectively.

Scheme of the breaking of the gauge group to the symmetric subgroup, which is like scheme of the Model 1, works for the Model 2 too. In this case vector-like multiplets $(\underline{5}, \underline{1}; \bar{\underline{5}}, \underline{1})$ from RNS-sector and $(\underline{1}, \underline{3}; \underline{1}, \underline{3})$ from $4b_3$ ($8b_3$) play the role of Higgs fields. Then generators of the symmetric subgroup and electromagnetic charges of particles are determined by formulas:

$$\begin{aligned} \Delta_{sym}^{(5)} &= t^{(5)} \times 1 \oplus 1 \times t^{(5)} \\ \Delta_{sym}^{(3)} &= (-t^{(3)}) \times 1 \oplus 1 \times t^{(3)} \\ Q_{em} &= t_5^{(5)} - 2/5 Y^5, \quad \text{where } t_5^{(5)} = (1/15, 1/15, 1/15, 2/5, -3/5) \end{aligned} \quad (67)$$

After this symmetry breaking matter fields (see Table 6) rows No 2, 3) standardly for flip models take place in representations of the $U(5)$ -group and form four generations $(\underline{1} + \underline{5} + \underline{10}; \bar{\underline{3}} + \underline{1})_{sym}$. And Higgs fields form adjoint representation of the symmetric group, similar to Model 1, which is necessary for breaking of the gauge group to the Standard group. Besides, quantization of the electromagnetic charge according to the formula (67) does not lead to appearance of exotic charges in lowenergy spectrum for this model too.

2.4 Superpotential and Non-renormalizable Contributions

The ability to correctly describe the fermion masses and mixings will, of course, constitute the decisive criterion for selection of a model of this kind. Therefore, within our approach one has to

1. study the possible nature of the G_H horizontal gauge symmetry ($N_g = 3_H$ or $3_H + 1_H$),

2. investigate the possible cases for G_H -quantum numbers for quarks (anti-quarks) and leptons (anti-leptons), i.e. whether one can obtain vector-like or axial-like structure (or even chiral $G_{HL} \times G_{HR}$ structure) for the horizontal interactions.
3. the structure of the sector of the matter fields which are needed for the $SU(3)_H$ anomaly cancelation (chiral neutral "horizontal" or "mirror" fermions),
4. write down all possible renormalizable and relevant non-renormalizable contributions to the superpotential W and their consequences for fermion mass matrices.

All these questions are currently under investigation. Here we restrict ourselves to some general remarks only.

With the chiral matter and "horizontal" Higgs fields available in the Model 1 constructed in this paper, the possible form of the renormalizable (trilinear) part of the superpotential responsible for fermion mass matrices is well restricted by the gauge symmetry:

$$\begin{aligned}
W_1 = & g\sqrt{2} \left[\hat{\Psi}_{(1,3)} \hat{\Psi}_{(\bar{5},1)} \hat{h}_{(5,\bar{3})} + \hat{\Psi}_{(1,1)} \hat{\Psi}_{(\bar{5},3)} \hat{h}_{(5,\bar{3})} + \right. \\
& \left. + \hat{\Psi}_{(10,3)} \hat{\Psi}_{(\bar{5},3)} \hat{h}_{(5,\bar{3})} + \hat{\Psi}_{(10,3)} \hat{\Psi}_{(10,1)} \hat{h}_{(5,\bar{3})} \right] \quad (68)
\end{aligned}$$

From the above form of the Yukawa couplings follows that two (chiral) generations have to be very light (comparing to M_W scale). The construction of realistic quarks and leptons mass matrices depends, of course, on the nature of the horizontal interactions. In the construction described in Sec.2.2 there is a freedom of choosing spin boundary conditions for $N_{LR}=12$ left and right fermions in the basis vectors b_3, b_5, b_6, \dots , which in the Ramond sector $2b_2$, may yield another Higgs fields, denoted as $\tilde{h}_{(\Gamma,N)}$ and transforming as $(\underline{5}, \underline{3})_{(-1,1)} + (\underline{5}, \underline{3})_{(1,-1)} \subset \underline{28} + \underline{28}$ of $SU(8)$. Using these Higgs fields we get the following alternative form of the renormalizable part of the superpotential W :

$$\begin{aligned}
W'_1 = & g\sqrt{2} \left[\hat{\Psi}_{(1,3)} \hat{\Psi}_{(\bar{5},3)} \tilde{h}_{(5,3)} + \hat{\Psi}_{(10,1)} \hat{\Psi}_{(\bar{5},3)} \tilde{h}_{(5,\bar{3})} + \right. \\
& \left. + \hat{\Psi}_{(10,3)} \hat{\Psi}_{(10,3)} \tilde{h}_{(5,3)} + \hat{\Psi}_{(10,3)} \hat{\Psi}_{(\bar{5},1)} \tilde{h}_{(5,\bar{3})} \right] \quad (69)
\end{aligned}$$

To construct the realistic fermion mass matrices one has to also use the Higgs fields (54, 55) and (Table 3, No 5) and also to take into account all relevant non-renormalizable contributions [20].

The Higgs fields (54) can be used for constructing Yukawa couplings of the horizontal superfields (No 3 and 4). The most general contribution of these fields to the superpotential is:

$$\begin{aligned}
W_2 = & g\sqrt{2} \left[\hat{\Phi}_{(1,1;1,\bar{3})}^H \hat{\Phi}_{(1,\bar{3};1,1)}^H \hat{\Phi}_{(1,3;1,3)} + \hat{\Phi}_{(1,1;1,1)}^H \hat{\Phi}_{(1,\bar{3};1,\bar{3})}^H \hat{\Phi}_{(1,3;1,3)} + \right. \\
& + \hat{\Phi}_{(1,\bar{3};1,\bar{3})}^H \hat{\Phi}_{(1,\bar{3};1,\bar{3})}^H \hat{\Phi}_{(1,\bar{3};1,\bar{3})} + \hat{\Psi}_{(1,3;1,1)}^H \hat{\Psi}_{(1,3;1,\bar{3})}^H \hat{\Phi}_{(1,3;1,3)} + \\
& \left. + \hat{\Psi}_{(1,1;1,\bar{3})}^H \hat{\Psi}_{(1,3;1,\bar{3})}^H \hat{\Phi}_{(1,\bar{3};1,\bar{3})} \right] \quad (70)
\end{aligned}$$

From this expression it follows that some of the horizontal fields in (63) (No 3, 4) remain massless at the tree-level. This is a remarkable prediction: fields (63) interact with the ordinary chiral matter fields only through the $U(1_H)$ and $SU(3_H)$ gauge boson and therefore are very interesting in the context of the experimental searches for the new gauge bosons.

The superfields $\hat{\Phi}_{(1,3;1,1)}^H$ and $\hat{\Psi}_{(1,\bar{3};1,1)}^H$ (see No 3, 4) can be used to construct the non-renormalizable contributions to the superpotential W . For example, the term

$$\Delta W_1 = \frac{cg^3}{M_{Pl}^2} \Psi_{(10,1)} \Psi_{(10,1)} \Phi_{(1,3;5,1)} \hat{\Phi}_{(1,3;1,1)}^H \hat{\Phi}_{(1,3;1,1)}^H \quad (71)$$

can give contribution to the mass to the fourth generation down-type quark ($c = \mathcal{O}(1)$, see [20]). To get a reasonable value of the mass for this quark we must arrange for the $SU(3_H)^I$ gauge symmetry breaking at the energy scale near the Planck scale, i.e. $\langle \hat{\Phi}_{(1,3;1,1)}^H \rangle = \langle \hat{\Phi}_{(1,\bar{3};1,1)}^H \rangle \sim M_{Pl}$. In this case one can get the $SU(3_H)^{II}$ -family gauge group with a low energy breaking symmetry scale. Finally, we remark that the Higgs sector of our GUST allows for conservation of the G_H gauge family symmetry down to the low energies ($\sim \mathcal{O}(1TeV)$ [9]). Thus we can expect at this energy region new interesting physics (new gauge bosons, new chiral matter fermions, superweak-like CP-violation in K ,- B ,- D -meson decays with $\delta_{KM} < 10^{-4}$ [9]).

3 Low Energy Construction of the $SU(3)_H$ model

3.1 The spontaneous breaking of SUSY $SU(3)_H$ horizontal gauge symmetry.

Since the expected scale of the horizontal symmetry breaking is sufficiently large: $M_H \gg M_{EW}$, $M_H \gg M_{SUSY}$ (where M_{EW} is the scale of the electroweak symmetry breaking, and M_{SUSY} is the value of the splitting into ordinary particles and their superpartners), it is reasonable to search for the SUSY-preserving stationary vacuum solutions.

Let us construct the gauge invariant superpotential P of Lagrangian (21). With the fields given in Table 11, the most general superpotential will have the form

$$\begin{aligned} P = & \lambda_0 \left[\frac{1}{3} Tr(\hat{\Phi}^3) + \frac{1}{2} M_I Tr(\hat{\Phi}^2) \right] + \lambda_1 \left[\eta \hat{\Phi} \xi + M' \eta \xi \right] + \lambda_2 Tr(\hat{h} \hat{\Phi} \hat{H}) + \\ & + \text{(Yukawa couplings)} + \text{(Majorana terms } \nu^c \text{)}, \end{aligned} \quad (72)$$

where Yukawa Couplings could be constructed, for example, using the Higgs fields, H and h , transforming under $SU(3)_H \times SU(2)_L$, like (8,2):

$$P_Y = \lambda_3 Q \hat{H} d^c + \lambda_4 L \hat{H} e^c + \lambda_5 Q \hat{h} u^c. \quad (73)$$

Also, one can consider another types of superpotential P_Y , using the Higgs fields from Table 11.

Table 1: **Basis of the boundary conditions for all world-sheet fermions. Model 1.**

Vectors	$\psi_{1,2}$	$\chi_{1,\dots,6}$	$y_{1,\dots,6}$	$\omega_{1,\dots,6}$	$\bar{\varphi}_{1,\dots,12}$	$\Psi_{1,\dots,8}$	$\Phi_{1,\dots,8}$
b_1	11	111111	111111	111111	1^{12}	1^8	1^8
b_2	11	111111	000000	000000	0^{12}	$1/2^8$	0^8
b_3	11	111100	000011	000000	$0^4 1^8$	0^8	1^8
$b_4 = S$	11	110000	001100	000011	0^{12}	0^8	0^8
b_5	11	001100	000000	110011	1^{12}	$1/4^5 - 3/4^3$	$-1/4^5 \ 3/4^3$
b_6	11	110000	000011	001100	$1^2 0^4 1^6$	1^8	0^8

Figure 2: Supersymmetry breaking.

$$\begin{array}{llll}
\text{N=2 SUSY :} & V & = & (1, \frac{1}{2}) + (\frac{1}{2}, 0) \quad SU(8) \\
\downarrow & & & \downarrow \\
\text{N=1 SUSY :} & V_{N=2} & \rightarrow & V_{N=1} + S_{N=1} \quad SU(5) \times SU(3) \times U(1)
\end{array}$$

	J=1	J=1/2	J=1/2	J=0
$E_{vac} = [-1/2; -1]$ NS sector	(63)	—	—	(63)
$E_{vac} = [0; -1]$ SUSY sector	—	$(63) \times 2$	$(63) \times 2$	—
	Gauge multiplets			

\downarrow b_5 projection GSO

	J=1	J=1/2	J=1/2	J=0
$E_{vac} = [-1/2; -1]$ NS sector	$(24,1) + (1,1) + (1,8)$	—	—	$(5, \bar{3}) + (\bar{5}, 3)$
$E_{vac} = [0; -1]$ SUSY sector	—	$((24,1) + (1,1) + (1,8)) \times 2$	$((5, \bar{3}) + (\bar{5}, 3)) \times 2$	—
	Gauge multiplets		Higgs multiplets	

Table 11. The Higgs Superfields with their $SU(3)_H, SU(3)_C, SU(2)_L, U(1)_Y$ (and possible $U(1)_H$ - factor) Quantum Numbers

	H	C	L	Y	Y_H
Φ	8	1	1	0	0
H	8	1	2	$-1/2$	$-y_{H1}$
h	8	1	2	$1/2$	y_{H1}
ξ	$\bar{3}$	1	1	0	0
η	3	1	1	0	0
Y	$\bar{3}$	1	2	$1/2$	$-y_{H2}$
X	3	1	2	$-1/2$	y_{H2}
κ_1	1	1	1	0	$-y_{H3}$
κ_2	1	1	1	0	y_{H3}

Note, that Higgs fields X and Y are very important in models with forth $SU(3)_H$ -singlet generation.

The spontaneous horizontal symmetry breaking may be constructed via different scenarios -both with intermediate scale, and without it:

$$\begin{aligned}
(i) \quad & SU(3)_H \xrightarrow{M_I} SU(2)_H \times U(1)_H \xrightarrow{M_H} c.b. \\
(ii) \quad & SU(3)_H \xrightarrow{M_I} U(1)_H \times U(1)_H \xrightarrow{M_H} c.b. \\
(iii) \quad & SU(3)_H \xrightarrow{M_I} U(1)_H \xrightarrow{M_H} c.b. \\
(iv) \quad & SU(3)_H \xrightarrow{M_{H0}} \text{complet breaking.}
\end{aligned} \tag{74}$$

If we assume that the soft breaking mass parameters in formula (22) should not be more than 0(1 TeV), then the soft breaking terms on the scale M_I of the $SU(3)_H$ - intermediate breaking may be neglected, and it is possible to go on working in the approximation of conserved SUSY. The SUSY preserving stationary vacuum solutions are degenerate in the models with global SUSY. In the construction of the stationary solutions, only the

Table 2: The choice of the GSO basis $\gamma[b_i, b_j]$. Model 1. (i numbers rows and j – columns)

	b_1	b_2	b_3	b_4	b_5	b_6
b_1	0	1	1	1	1	0
b_2	1	$1/2$	0	0	$1/4$	1
b_3	1	$-1/2$	0	0	$1/2$	0
b_4	1	1	1	1	1	1
b_5	0	1	0	0	$-1/2$	0
b_6	0	0	0	0	1	1

Table 3: **The list of quantum numbers of the states. Model 1.**

N ^o	$b_1, b_2, b_3, b_4, b_5, b_6$	SO_{hid}	$U(5)^I$	$U(3)^I$	$U(5)^{II}$	$U(3)^{II}$	Y_5^I	Y_3^I	Y_5^{II}	Y_3^{II}
1	RNS 0 2 0 1 2(6) 0		5	3	1	1	-1	-1	0	0
			1	1	5	$\bar{3}$	0	0	-1	-1
			5	1	5	1	-1	0	-1	0
			1	3	1	3	0	1	0	1
			5	1	1	3	-1	0	0	1
			1	3	5	1	0	1	-1	0
2	0 1 0 0 0 0 0 3 0 0 0 0		1	3	1	1	5/2	-1/2	0	0
			$\bar{5}$	3	1	1	-3/2	-1/2	0	0
			10	1	1	1	1/2	3/2	0	0
			1	1	1	1	5/2	3/2	0	0
			$\bar{5}$	1	1	1	-3/2	3/2	0	0
			10	3	1	1	1/2	-1/2	0	0
3	0 0 1 1 3 0	$-1 \pm_2$	1	1	1	3	0	-3/2	0	-1/2
	0 0 1 1 7 0	$-1 \pm_2$	1	$\bar{3}$	1	1	0	1/2	0	3/2
	0 2 1 1 3 0	$+1 \pm_2$	1	$\bar{3}$	1	3	0	1/2	0	-1/2
	0 0 1 1 7 0	$+1 \pm_2$	1	1	1	1	0	-3/2	0	-3/2
4	1 1 1 0 1 1	$\mp_1 \pm_3$	1	1	1	3	0	-3/2	0	1/2
	1 1 1 0 5 1	$\mp_1 \pm_3$	1	$\bar{3}$	1	1	0	1/2	0	-3/2
	1 3 1 0 1 1	$\pm_1 \pm_3$	1	$\bar{3}$	1	$\bar{3}$	0	1/2	0	1/2
	1 3 1 0 5 1	$\pm_1 \pm_3$	1	1	1	1	0	-3/2	0	-3/2
5	0 1(3) 1 0 2(6) 1	$-1 \pm_3$	1	3(3)	1	1	$\pm 5/4$	$\pm 1/4$	$\pm 5/4$	$\mp 3/4$
		$+1 \pm_3$	5($\bar{5}$)	1	1	1	$\pm 1/4$	$\mp 3/4$	$\pm 5/4$	$\mp 3/4$
	0 1(3) 1 0 4 1	$-1 \pm_3$	1	1	1	3($\bar{3}$)	$\pm 5/4$	$\mp 3/4$	$\pm 5/4$	$\pm 1/4$
		$+1 \pm_3$	1	1	5($\bar{5}$)	1	$\pm 5/4$	$\mp 3/4$	$\pm 1/4$	$\mp 3/4$
6	1 2 0 0 3(5) 1	$\pm_1 -_4$	1	1	1	1	$\pm 5/4$	$\pm 3/4$	$\mp 5/4$	$\mp 3/4$
	1 1(3) 0 1 5(3) 1	$+1 \mp_4$	1	1	1	1	$\pm 5/4$	$\pm 3/4$	$\pm 5/4$	$\pm 3/4$
	0 0 1 0 2(6) 0	$\mp_3 +_4$	1	1	1	1	$\pm 5/4$	$\mp 3/4$	$\pm 5/4$	$\mp 3/4$

following contributions of the scalar potential are taken into account:

$$V = \sum_i |F_i|^2 + \sum_a |D^a|^2 = V_F + V_D \geq 0 \quad (75)$$

$$\text{where } V_F = \sum \left| \frac{\partial P_F}{\partial F_i} \right|^2 = \left| \frac{\partial P_F}{\partial F_{\Phi^a}} \right|^2 + \left| \frac{\partial P_F}{\partial F_{\xi_i}} \right|^2 + \left| \frac{\partial P_F}{\partial F_{\eta_i}} \right|^2 \quad (76)$$

The case $\langle V \rangle = 0$ of supersymmetric vacuum can be realized within different gauge scenarios (74). By switching on the SUGRA, the vanishing scalar potential is no more required to conserve the supersymmetry with the necessity. Hence, different gauge breaking scenarios (74) do not result in obligatory vacuum degeneracy, as in the case of the global SUSY version. Let us write down each of the terms of formula (76):

$$\begin{aligned}
 P_F(\Phi, \xi, \eta) &= \lambda_0 \left[\frac{i}{4 \times 3} f^{abc} \Phi^a \Phi^b \Phi^c + \frac{1}{4 \times 3} d^{abc} \Phi^a \Phi^b \Phi^c + \frac{1}{4} M_I \Phi^c \Phi^c \right]_F + \\
 &+ \lambda_1 \left[\eta_i (T^c)_j^i \xi^j \Phi^c + M' \eta_i \xi^i \right]_F + \quad (77)
 \end{aligned}$$

$$+ \lambda_2 \left[\frac{i}{4} f^{abc} h_i^a \Phi^b H_j^c \epsilon^{ij} + \frac{d^{abc}}{4} h_i^a \Phi^b H_j^c \epsilon^{ij} \right]_F + h.c.$$

The contribution of D -terms into the scalar potential will be :

$$\begin{aligned} V_D = & g_H^2 |\eta^+ T^a \eta - \xi^+ T^a \xi + i/2 f^{abc} \Phi^b \Phi^{c+} + i/2 f^{abc} h^b h^{c+} + i/2 f^{abc} H^b H^{c+}|^2 \\ & + g_2^2 |h^+ \tau^i / 2 h + H^+ \tau^i / 2 H|^2 + (g')^2 |1/2 h^+ h - 1/2 H^+ H|^2 \end{aligned} \quad (78)$$

The SUSY-preserving condition for scalar potential (75) is determined by the flat F_i - and D^a directions: $\langle F_i \rangle_0 = \langle D^a \rangle_0 = 0$. It is possible to remove the degeneracy of the supersymmetric vacuum solutions taking into account the interaction with supergravity, which was endeavored in SUSY GUT's, e.g. in the $SU(5)$ one [31] ($SU(5) \rightarrow SU(5), SU(4) \times U(1), SU(3) \times SU(2) \times U(1)$).

The horizontal symmetry spontaneous breaking to the intermediate subgroups in the first three cases of (74) can be realized, using the scalar components of the chiral complex superfields Φ , which are singlet under the standard gauge group. The Φ -superfield transforms as the adjoint representation of $SU(3)_H$. The intermediate scale M_I can be sufficiently large: $M_I > 10^5 - 10^6 \text{ GeV}$. The complete breaking of the remnant symmetry group V_H on the scale M_H will occur due to the nonvanishing VEV's of the scalars from the chiral superfields $\eta(3_H)$ and $\xi(\bar{3}_H)$. The V_{min} , again, corresponds to the flat directions: $\langle F_{\eta, \xi} \rangle_0 = 0$. The version (iv) corresponds to the minimum of the scalar potential in the case when $\langle \Phi \rangle_0 = 0$.

As for the electroweak breaking, it is due to the VEV's of the fields h and H , providing masses for quarks and leptons. Note that VEV's of the fields h and H must be of the order of M_W as they determine the quark and lepton mass matrices. On the other hand, the masses of physical Higgs fields h and H , which mix generations, must be some orders higher than M_W , so as not to contradict the experimental restrictions on FCNC. As a careful search for the Higgs potential shows, this is the picture that can be attained.

3.2 The intermediate horizontal symmetry breaking

As noted in the previous Section, the spontaneous horizontal gauge symmetry breaking takes place when the fields ϕ , η and ξ get nonvanishing VEVs. We are interested in the possibility of realizing the structure, when some of the horizontal gauge bosons (and the corresponding gauginos) may have relatively small masses ($M_H \sim 1 - 10 \text{ TeV}$) [9]. Our consideration of the family symmetry breaking will be done in two steps. To this end, we look for the SUSY stationary vacuum solutions, such as $\langle \Phi \rangle_0 \gg \langle \eta \rangle_0, \langle \xi \rangle_0$. So, the degeneracy of the corresponding H -gauge bosons is assumed near one or two scales. The complete breaking of the $SU(3)_H$ - group corresponds to the "condensation" of all eight bosons near the M_H scale. For intermediate $SU(3)_H$ - breakings, some of the gauge massive superfields will have the masses around the scale M_I , while the other superfields from the remnant symmetry group will be condensed on the scale M_H ($M_H \ll M_I$). We will analyze several subgroups of $SU(3)_H$ - and check if the low scale M_H is consistent with the experimental data for these models. Such analysis will allow us to

get a deeper insight into the dynamics of horizontal forces and investigate the effects of their compensation, especially in pure leptonic and pure quark processes. At first stage, due to the nonvanishing VEV of Φ , the horizontal symmetry group breaks down to some subgroup V satisfying $[V, \langle \Phi \rangle_0] = 0$. At the second stage, the remnant group V is broken down completely, as fields η and ξ will acquire nonzero VEVs. Let us consider several cases of this breaking.

Case (i): $V = SU(2)_H \times U(1)_{8H}$. As has already been mentioned, in the gauge model with the global SUSY stationary supersymmetry conserving vacuum solutions are degenerate: $V_{min} = 0$. Let us recall that the superinvariance condition for the model on the scale M_I requires the existence of flat D^a - and F_Φ^a - directions: $\langle D^a \rangle_0 = \langle F_\Phi^a \rangle_0 = 0$ ($a = 1, 2, 3, 8$). Equations (76-78) give the following form of these constraints:

$$1/2 d^{abc}(\Phi_1^a \Phi_1^b - \Phi_2^a \Phi_2^b) + M_I \Phi_1^c = 0 \quad (\langle F_\Phi \rangle_0 = 0) \quad (79)$$

$$d^{abc} \Phi_1^a \Phi_2^b + M_I \Phi_2^c = 0$$

$$i f^{abc} \Phi^b \Phi^{c+} = 0 \quad (\langle D^a \rangle_0 = 0), \quad (80)$$

where $\Phi^a = \Phi_1^a + i\Phi_2^a$, d^{abc} and f^{abc} are the $SU(3)$ structure constants. From equations (79) and (80) it is easy to verify that the SUSY $SU(3)_H$ - group can be broken down to the SUSY $SU(2)_H \times U(1)_{8H}$ if, for example, the 8-th component of the field Φ acquires a nonvanishing VEV:

$$\langle \Phi^8 \rangle_0 = \frac{\sqrt{3}a_8}{2} = \frac{\sqrt{3}M_I}{2} \quad (81)$$

In this case of the gauge symmetry breaking the supersymmetry conservation allows to describe the mass spectrum of new massive $N = 1$ supermultiplets in a rather simply way. We start with eight vector massless superfields $V_H^a(1, 1/2)$ ($4 \times 8^a = 32$ degrees of freedom) and eight chiral massless superfields $\Phi^a(1/2; 0, 0)$ ($4 \times 8^a = 32$ degrees of freedom). As a result of the super-Higgs effect, we get four massive vector supermultiplets $(1, \frac{1}{2}) + (\frac{1}{2}, 0 + 0) = (1, \frac{1}{2} + \frac{1}{2}, 0)_{massive}$ with $8 \times 4^a = 32$ degrees of freedom and with the same universal mass. The formula for the gauge boson mass is

$$\begin{aligned} (M^2)_{ab} &= 1/2 g_H^2 f^{8ac} f^{8bc} a_8^2 = 3/8 g_H^2 a_8^2 \delta^{ab} \\ a, b &= 4, 5, 6, 7 \quad or \\ M_{4,5,6,7}^2 &= 3/8 g_H^2 M_I^2, \quad M_{1,2,3,8}^2 = 0 \end{aligned} \quad (82)$$

The mass term of λ -gauginos is expressed as follows:

$$\begin{aligned} \mathcal{L}_M &= 1/\sqrt{2} g_H f^{8bc} \psi_\Phi^b \lambda^c a_8 = \frac{\sqrt{3} g_H}{2 \sqrt{2}} M_I [\psi_\Phi^4 \lambda^5 - \psi_\Phi^5 \lambda^4 + \psi_\Phi^6 \lambda^7 - \psi_\Phi^7 \lambda^6] \\ &- \lambda_0 M_I 3/4 (\psi_\Phi^1 \psi_\Phi^1 + \psi_\Phi^2 \psi_\Phi^2 + \psi_\Phi^3 \psi_\Phi^3 - 1/3 \psi_\Phi^8 \psi_\Phi^8) + h.c. \end{aligned} \quad (83)$$

So the gauginos $\lambda^4, \lambda^5, \lambda^6, \lambda^7$ combining with fermions $\psi_\Phi^4, \psi_\Phi^5, \psi_\Phi^6, \psi_\Phi^7$ give the Dirac gauginos with the masses $M = \frac{1}{\sqrt{2}} \frac{\sqrt{3}}{2} g_H M_I$. Four real scalar states from the supermultiplets $\Phi^{4,5,6,7}$ transform into the longitudinal components of four corresponding massive vector bosons, while the remaining four scalar states contribute to four massive $N = 1$ supermultiplets $(1, \frac{1}{2} + \frac{1}{2}, 0)_{\text{massive}}$. There are also four massless vector superfields $V_H^{1,2,3,8}$ (16 degrees of freedom) and four massive chiral superfields (ψ_Φ^a, Φ^a) ($a = 1, 2, 3, 8$) at this stage of breaking. So, due to the super-Higgs mechanism of SUSY breaking four massless vector superfields have absorbed four massless chiral superfields and formed four massive vector superfields. The chiral superfields $\Phi^{4,5,6,7}$ play the role of Higgs superfields and they all have been absorbed completely.

At the second stage of the $SU(2)_H \times U(1)_{8H}$ gauge symmetry breaking with a simultaneous supersymmetry conservation, one can use the chiral superfields η_α, ξ_α with equal VEV's: $\langle \eta_{\alpha i} \rangle_0 = \langle \xi_\alpha^i \rangle_0 = \delta_\alpha^i \gamma$ ($i, \alpha = 1, 2, 3, 8$). As a result of this breaking, four massive vector supermultiplets $V_H^{1,2,3,8}(1, \frac{1}{2} + \frac{1}{2}, 0)$ acquire the universal mass $M_H^2 = 2g_H^2 \gamma^2$. A detailed analysis shows that the Majorana higgsinos from the supermultiplets η_α, ξ_α participate in the formation of four Dirac gauginos, whose upper components are $\lambda^1, \lambda^2, \lambda^3$ and λ^8 . The degenerate mass of these Dirac gauginos will be $\sqrt{2}g_H \gamma$:

$$\begin{aligned} \mathcal{L}_M = & i/\sqrt{2} g_H \gamma \left\{ \lambda^3 [(\eta_{11} - \xi_1^1) - (\eta_{22} - \xi_2^2)] + 1/\sqrt{3} \lambda^8 [(\eta_{11} - \xi_1^1) + \right. \\ & + (\eta_{22} - \xi_2^2) - 2(\eta_{33} - \xi_3^3)] + \lambda^1 [(\eta_{21} - \xi_1^2) + (\eta_{12} - \xi_2^1)] \\ & \left. - i\lambda^2 [(\eta_{12} + \xi_2^1) + (\eta_{21} + \xi_1^2)] \right\} + h.c. \end{aligned} \quad (84)$$

It is easy now to rewrite the Lagrangian of the interactions in terms of physical states (remembering that for the matter fields $\psi_{mi} = U_{ij} \psi_{oj}$ and $A_{mi} = \tilde{U}_{ij} A_{oj}$, where A denotes the scalar partner of ψ -fermions). The gauge boson interactions with matter fields have the form:

$$\begin{aligned} \mathcal{L} = g_H H_a^\mu \left\{ \bar{\psi}_u \gamma_\mu U_L T^a U_L^+ \frac{1 + \gamma_5}{2} \psi_u + \bar{\psi}_u \gamma_\mu U_R T^a U_R^+ \frac{1 - \gamma_5}{2} \psi_u + \right. \\ \left. + (u \rightarrow d, l, \nu) \right\} \end{aligned} \quad (85)$$

Let us consider now the gaugino interactions. The initial Lagrangian has the form:

$$\mathcal{L} = ig_H \sqrt{2} (A_i^+ T_{ij}^a \lambda^a \psi_j - h.c.) \quad a = 1, 2, 3, 8 \quad (86)$$

Consider only the interaction between left "up" quarks, left "up" squarks and gauginos. The generalization of this Lagrangian to all leptons and quarks will be obtained by simply adding similar terms to u_R, d, d_R, ν, l and l_R . Expression (86) for "up" quarks looks like:

$$\begin{aligned} \mathcal{L} = & ig_H (A_1^*, A_2^*, A_3^*)_L \begin{pmatrix} \lambda^3 + \frac{1}{\sqrt{3}} \lambda^8 & \lambda^+ & 0 \\ \lambda^- & -\lambda^3 + \frac{1}{\sqrt{3}} \lambda^8 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} \lambda^8 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} + h.c. = \\ = & -g_H [A_{iL}^* \tilde{U}_{ik} (\Lambda')_{bk} U_{jb}^* \frac{1 + \gamma_5}{2} \psi_j] + h.c. \end{aligned} \quad (87)$$

where $A_{iL} = \tilde{U}_{ij} A_{0jL}$, $i, j, \dots, b = 1, 2, 3$.

Case (ii). To realize this version of the intermediate $SU(3)_H$ - symmetry breaking, one has to use the pair of the chiral superfields $\Phi, \tilde{\Phi}$ with different $U(1)_R$ quantum numbers. Then one easily verifies that the stationary supersymmetric vacuum solutions will be realized in accordance with equations (75-77). These solutions will look like

$$\langle \Phi_1^3 \rangle_0 = \langle \Phi_1^8 \rangle_0 = -\sqrt{3}M, \quad \langle \tilde{\Phi}_1^8 \rangle_0 = 2\sqrt{3}M \quad (88)$$

When these fields are applied simultaneously with the above VEVs (88), the following gauge boson mass spectrum is obtained: $M_{H_4} = M_{H_5} = 3\sqrt{2}M_I$, $M_{H_1} = M_{H_2} = M_{H_6} = M_{H_7} = 3M_I$, $M_{H_3} = M_{H_8} = 0$. The remnant group in this case is the $SUSY U(1)_{3H} \times U(1)_{8H}$ - group. As one would expect, the rank of the group did not change, whereas the remnant group was broken by the chiral superfields η, ξ on the scale M_H . Here it makes no difficulty to get the mass degeneracy of the superfields V_3 and V_8 conserving SUSY while doing this. Again, the super-Higgs mechanism is applied leading to the formation of the massive superfields with the universal mass M_H . In this connection, a rather simple way may be proposed to estimate the bound on M_H from the comparison with the data on rare processes.

And, finally, let us consider case (iii) when $V = U(1)_{8H}$. We confine ourselves to the case when the scalar components of the complex chiral superfield $\tilde{\Phi}$ have the nonzero VEVs: $\langle \Phi_1^1 \rangle_0 \neq 0$, $\langle \Phi_1^2 \rangle_0 \neq 0$, $\langle \Phi_1^3 \rangle_0 \neq 0$, $\langle \Phi_1^8 \rangle_0 \neq 0$.

Although this choice of VEV's fulfils the equations for the flat F_Φ directions with the solutions $\langle \Phi_1^8 \rangle_0 = -\sqrt{3}M_I$, $\langle \Phi_1^1 \rangle_0^2 + \langle \Phi_1^2 \rangle_0^2 - \langle \Phi_1^3 \rangle_0^2 = 9M_I^2$, this solution does not determine the vacuum of the theory as might be expected. The corresponding solutions for $D^{1,2}$ are incompatible with the F_Φ -flat solutions. As in the previous case (ii), in order to overcome this difficulty one has to introduce a new Higgs superfield to compensate for the nonvanishing contributions of D -terms to the scalar potential of the theory. This compensation requires a specific choice of the vacuum expectations for the second Higgs superfield $\tilde{\Phi}$. In this case, only one vector supermultiplet V_H^8 is left on the intermediate scale.

The abovementioned examples are enough to research further into the regularity of the behavior of the violation scale M_H by comparing model predictions with the experiment. Here we just note that the SUSY stationary solutions with CP violation in the horizontal sector are available both for the scales M_I and M_H . Indeed, for instance, in case (iii) the CP violation occurs on the scale M_I . Another supersymmetric vacuum, $\langle \Phi_1^8 \rangle_0 = -\sqrt{3}M_I$, $\langle \Phi_1^1 \rangle_0^2 + \langle \Phi_1^2 \rangle_0^2 + \langle \Phi_1^3 \rangle_0^2 = 9M_I^2$, corresponding to the intermediate symmetry group $SU(2)_H \times U(1)_{8H}$, results in the CP violation in the neutral K -meson decays due to only horizontally acting forces on the scale M_H . In the last case, in the electroweak sector of CP violation one may have $\delta_{KM} = 0$. That is the very case outlined in our introduction.

3.3 The role of horizontal interactions with intermediate symmetry breaking scale in rare processes

Let us analyze the contribution of horizontal interactions to rare processes. We will consider first the oscillations of K^0 , B_d^0 and B_s^0 mesons. The experimental data on $K^0 \longleftrightarrow \bar{K}^0$ oscillations are as follows:

$$\left(\frac{\Delta m_K}{m_K}\right)_H < \left(\frac{\Delta m_K}{m_K}\right)_{exp} \approx 7 \times 10^{-15} \quad (89)$$

The theoretical expression for the $m(K_L) - m(K_S) = \Delta m_K$ mass difference is given by the equation:

$$\Delta m_K = \frac{g_H^2}{M_H^2} \text{Re}(C_K^0) f_K^2 m_K \left[\frac{1}{6} + \frac{1}{3} \frac{m_K^2}{(m_s - m_d)^2} \right], \quad (90)$$

where $C_K^0 = \sum'_a (DT^a D^+)_{21} (DT^a D^+)_{21}$, C_K^0 being a unitary coefficient showing the contributions of the Feynman diagrams with the exchange of the horizontal bosons from the considered gauge groups. The symbol "r" denotes that the sum is over the definite set of indexes "a", but it should be noted that the sum over the complete set (a=1,2,...8) is equal to zero. "D" is the orthogonal matrix diagonalizing the Fritzsch-like mass matrix for "down" quarks. One-particle contributions to the vector, axial, scalar and pseudoscalar currents might be calculated like in Ref. [36].

From formula (90), using the experimental data and the values for $\alpha_H = g_H^2/4\pi \approx 1.9 \cdot 10^{-2}$ on the scale M_H [9], one can obtain the lower limits on the light H-boson masses. These values are given in Table 12 together with the C_K^0 values.

Analogous expressions can be obtained for the B_d and B_s mesons. The mixing elements for B_d and B_s mesons are as follows: $C_{B_d}^0 = \sum'_a (DT^a D^+)_{31} (DT^a D^+)_{31}$ and $C_{B_s}^0 = \sum'_a (DT^a D^+)_{32} (DT^a D^+)_{32}$. Their values are given in Table 12. Using the H-boson mass limits from Table 12 and the value $f_{B_d} \approx 150$ MeV, one can calculate the B_d meson mass difference (see Table 12). The one-particle contribution (R_1) to the B_d (and B_s) meson amplitudes is unknown. But, assuming that it is not much greater than the vacuum contribution, one can see that the $\Delta m_{B_d}/m_{B_d}$ values given in Table 12 are very close to those obtained from ARGUS [5] (except for case (ii)):

$$\left(\frac{\Delta m_{B_d}}{m_{B_d}}\right)_H < \left(\frac{\Delta m_{B_d}}{m_{B_d}}\right)_{exp} = (0.73 \pm 0.14) \times 10^{-13} \quad (91)$$

Table 12. The $M_H, C_K^0, C_{B_d}^0, C_{B_s}^0, \Delta m_{B_d}/m_{B_d}$ and $\Delta m_{B_s}/\Delta m_{B_d}$ Values for Different Models.

Models	C_K^0	M_H (TeV)	$C_{B_d}^0$	$\frac{\Delta m(B_d)}{m(B_d)}$	$\frac{\Delta m(B_s)}{\Delta m(B_d)}$
$SU(2)_H \times U(1)_{8H}$	3.8×10^{-5}	$8 \div 9$	1.5×10^{-3}	$(1.1 \div 0.8) \times 10^{-13}k$	m_s/m_d
$U(1)_{3H} \times U(1)_{8H}$	1.6×10^{-2}	$170 \div 200$	1.5×10^{-3}	$(2.3 \div 1.8) \times 10^{-16}k$	m_s/m_d
$U(1)_{8H}$	2.8×10^{-5}	$7 \div 8$	1.1×10^{-3}	$(1 \div 0.8) \times 10^{-13}k$	m_s/m_d
$SU(3)_H$	$< (10^{-5} \div 10^{-6})$	$O(1 \text{ TeV})$	—	—	—

$k = (R_{1P} + 1/2)$, $g_H \approx g_{EW}$.

Let us calculate now the relative mass difference of mesons B_d and B_s . Assuming that $f_{B_s} \approx f_{B_d}$, one obtains the values given in Table 12. So the oscillations of B_s mesons in such models must be stronger than those of B_d mesons. Let us consider now the decay $\mu \rightarrow 3e$. The branching ratio of this process will be

$$B(\mu \rightarrow 3e) = 12 \frac{g_H^4}{g_W^4} \frac{m_W^4}{M_H^4} |C_{(\mu)}^0|^2, \quad (92)$$

where $C_{(\mu)}^0 = \sum_a' (LT^a L^+)_{21} (LT^a L^+)_{11} = \sum_a' L_{21}^a L_{11}^a$; L- is the orthogonal matrix diagonalizing the real Fritzsche- like mass matrix for "down" leptons. Using the experimental value [37] $B(\mu \rightarrow 3e) < 10^{-12}$, it is easy to obtain the limits for the horizontal boson masses:

$$\begin{aligned} (i) \quad M_H &> 7.5 \text{ TeV} & (ii) \quad M_H &> 30 \text{ TeV} \\ (iii) \quad M_H &> 5 \text{ TeV} & (iv) \quad M_H &> O(1 \text{ TeV}). \end{aligned} \quad (93)$$

Let us turn next to the process of the muon-to-electron conversion in the presence of a nucleus. The branching ratio of this process for the nucleus with equal numbers of protons and neutrons and large Z is [28]:

$$\frac{\Gamma(\mu N \rightarrow e N)}{\Gamma(\mu N \rightarrow \nu N)} = 432 \frac{|\sum_a' L_{21}^a [U_{11}^{a*} + D_{11}^{a*}]|^2}{1/4 (1 + 3g_A^2)} \frac{m_W^4}{M_H^4} \frac{g_H^4}{g_W^4}. \quad (94)$$

In eq.(94), L_{21}^a, U_{11}^a and D_{11}^a are the mixing elements for leptons, up- and down-quarks. Using the recent experimental value for the μ to e conversion : $\Gamma(\mu N \rightarrow e N) < \Gamma(\mu N \rightarrow \nu N) \times 5 \cdot 10^{-12}$ [38], from eq. (94) one can obtain the limits for the horizontal gauge bosons masses.

We consider the choice for the forms of the quark and lepton mass matrices, for instance, the "improved" Fritzsche ansatz like Matumoto [11], the corresponding estimates will do not change much (except for $U(1)_{8H}$)

$$\begin{aligned} (i) \quad M_H &> 60 \text{ TeV} & (ii) \quad M_H &> 65 \text{ TeV} \\ (iii) \quad M_H &> 3 \text{ TeV} & (iv) \quad M_H &> 60 \text{ TeV}. \end{aligned} \quad (95)$$

This fact can easily be explained by the coincidence of the values of the mass matrix element $(M_d)_{12}$ in these two ansatzes and its dominant role in the definition of the V_{us} -CKM matrix element.

From another very important quark-lepton rare decay $K^+ \longrightarrow \pi^+ \mu^+ e^-$, whose partial width is now experimentally estimated as

$$Br(K^+ \longrightarrow \pi^+ \mu^+ e^-) < 2.1 \times 10^{-10}, \text{BNL} - E777, \quad (96)$$

the constraints on M_{H_0} are also rather large (except in (iii)):

$$M_{H_0} > g_H/g_W \times 35 TeV. \quad (97)$$

Let us compare it with the bounds on the pure quark or lepton rare processes. For the $U(1)_{8H^-}$ group, the corresponding bound on the scale M_{H_0} is approximately some TeVs.

Finally, let us consider the decay $\mu \rightarrow e\gamma$. The one-loop contribution with the H -boson exchange is suppressed against the $\mu \rightarrow 3e$ decay: $\Gamma(\mu \rightarrow e\gamma) \ll \Gamma(\mu \rightarrow 3e)$. So, the major contribution to the $\mu \rightarrow e\gamma$ decay width will come from the one-loop diagram with the exchange of horizontal gauginos and scalar charged leptons. The branching ratio of this decay is :

$$B(\mu \rightarrow e\gamma) = \frac{48\pi^2}{G_F^2 m_\mu^2} F^2 \quad (98)$$

, where formfactor F^2 is given in ref.[28]. Using the experimental value cite38': $B(\mu \rightarrow e\gamma) < 4.9 \cdot 10^{-11}$, one can easily obtain the bounds on the gaugino masses:

$$\begin{aligned} (i) \quad \tilde{M}_H &> 0.6 TeV & (ii) \quad \tilde{M}_H &> 0.25 TeV \\ (iii) \quad \tilde{M}_H &> 0.3 TeV & (iv) \quad \tilde{M}_H &> O(100 GeV), \end{aligned} \quad (99)$$

where the scalar lepton mass is 80 GeV.

To conclude, let us note that the analysis of the supersymmetric horizontal model shows that in several schemes of H -symmetry breaking (cases (i), (iii) and (iv)) the limits for the lower bounds of some H -bosons from the experimental results on the amplitudes of pure quark and pure leptonic rare processes ($|\Delta H| \neq 0$) can be relatively low ($\leq 10 TeV$). In this case the contribution of H -interaction to B_d^0 meson oscillations may turn out to be the major contribution and explain, in principle, the experimental value of the $B_{d1}^0 - B_{d2}^0$ mass difference. However, similar bounds on M_H , derived from some quark-lepton rare reactions ($|\Delta H| = 0$), may turn out to be much more than the above estimates, except for case (iii).

Really, we should look closer at this situation : in particular, we should clear up whether our understanding of the origin of quark and lepton generations is correct, i.e. that we see one and the same quark and lepton mixing mechanism in operation. Indeed, by now no reliable evidences to the lepton mixing have been obtained. It is natural to think that the problem of the nature of mixings ascends to the main question of the SM, i.e. the origin of quark and lepton masses. So we state that we know nothing about how the mixings of quark and lepton families are correlated and can only hypothesize it.

Before considering a particular model of such a nontrivial correlation and discussing some of its consequences for the breaking scale of the horizontal gauge interaction M_H , we have to investigate the breaking of the $SU(3)_H$ -model without the intermediate scale M_{H_I} , trying to connect the splitting of its global breaking scale M_{H_0} with known heavy quark masses. This will enable us to get more information about the 8-gauge boson masses and make our estimates more predictive.

4 The SUSY $SU(3)_H$ -gauge model with correlation 3- family mixing and 8-gauge boson mass splitting.

In this Section we will confine ourselves to the consideration of two types of Hermitian fermion mass matrices and, going on with the previous analysis, estimate the local H-symmetry breaking scale (see subsection 1.4)

As the 1st approach, we consider a modified "calculable" Fritzsch ansatz for 3 generations, with nonzero antidiagonal mass matrix elements and the upper bound on the t-quark from the experimental data on the matrix element V_{cb} will be in according to the experimental values.

$$\mathbf{M}_f^{\mathbf{MF}} = \begin{pmatrix} 0 & Ae^{-i\alpha} & Ee^{-i\gamma} \\ Ae^{i\alpha} & F & Be^{-i\beta} \\ Ee^{i\gamma} & Be^{i\beta} & C \end{pmatrix}, \quad (100)$$

where $(A \gg \text{or} \sim E) \ll (B \ll \text{or} \sim F) \ll C$. We will not study now the most general form of hermitian quark mass matrices of the three families. It will be enough, and more useful, for us to consider the specific forms of quark ansatzes which would fit well the CKM- mixing matrix elements with the experimental accuracy attainable today.

Another (democratic) ansatz is noteworthy for the possibility to single out the t-quark mass value. And besides, within this approach the mass matrices of the above form can, at least, be more correctly interpreted in physical terms - e.g., via the compositeness of quarks.

4.1 The modified Fritzsch ansatz

Let us consider some of the "calculable" ansatzes for 3×3 "up" and "down" quark mixing matrices with nonzero antidiagonal elements (100) and consistent with the modern values of the CKM-matrix for charged currents [11]c, where the matrix elements of the up- and down mass matrices were taken like as:

$$\begin{aligned} A_u &= E_u = \sqrt{|m_u|m_c}, F_u = -B_u = m_c, C = m_t, \\ \alpha_u &= \beta_u = \gamma_u = 0; \\ A_d &= E_d = \sqrt{|m_d|m_s}, F_d = B_d = m_s, C_d = m_b, \\ \alpha_d &= -\frac{\pi}{2}, \beta_d = \gamma_d = 0. \end{aligned} \quad (101)$$

In the leading approximation the unitary matrices U and D ($O_{CKM} = U D^+$), diagonalizing these mass states (100) (101), have the following form:

$$U \approx \begin{pmatrix} 1 & -\sqrt{\frac{m_u}{m_c}} & -2\sqrt{\frac{m_u m_c}{m_c m_t}} \\ \sqrt{\frac{m_u}{m_c}} & 1 & \frac{m_c}{m_t} \\ \sqrt{\frac{m_u m_c}{m_c m_t}} & -\frac{m_c}{m_t} & 1 \end{pmatrix} \quad (102)$$

and

$$\mathbf{D} \approx \begin{pmatrix} e^{-i\frac{\pi}{4}} & \sqrt{\frac{m_d}{m_s}} e^{-i\frac{3\pi}{4}} & \frac{\sqrt{2m_d m_s}}{m_b} e^{i\frac{\pi}{2}} \\ \sqrt{\frac{m_d}{m_s}} & e^{i\frac{\pi}{2}} & \frac{m_s}{m_b} e^{-i\frac{\pi}{2}} \\ \frac{\sqrt{m_d m_s}}{m_b} e^{i\frac{\pi}{2}} & \frac{m_s}{m_b} e^{i\frac{\pi}{2}} & e^{i\frac{\pi}{2}} \end{pmatrix}. \quad (103)$$

Using the above ansatz for the $b \rightarrow c$ transition, one can get a higher restriction for the upper bound on the t- quark mass: $V_{cb} \approx \frac{m_s}{m_b} + \frac{m_c}{m_t}$.

The experimental precision of the measurements of V_{cb} could indicate, and only to a certain extent, the magnitude of the d_{23} . Unfortunately, our knowledge of the d_{13} - element is still not sufficient because of big experimental uncertainties in q or $\sqrt{\rho^2 + \eta^2}$ values ($q \leq 0.2$). In scheme [11]c $q = |V_{ub}|/|V_{cb}| \approx 0.1$ and the Wolfenstein parameters are $\rho \approx -0.15$ and $\eta \approx 0.49$ ($\sqrt{\rho^2 + \eta^2} \approx \frac{1}{2}$), so that the $(V_{ub} - V_{td})$ - intersection of the unitary triangle lies in the second (left) quadrant of the ρ, η - complex plane.

To obtain the form of these mass matrices, it's necessary to consider, besides H - and h -Higgs superfields (see (72)), the additional superfields $H_0(1_H, 2_L, -\frac{1}{2})$ and $h_0(1_H, 2_L, \frac{1}{2})$, which are $SU(3)_H$ -singlets. Then the corresponding addenda $k_1 Q h_0 u^c$ and $k_2 Q H_0 d^c$ will appear in the superpotential (72).

The splitting between the horizontal gauge boson masses will be determined only by the nonvanishing VEV's of the $H(8_H, 2_L, -\frac{1}{2})$ and $h(8_H, 2_L, +\frac{1}{2})$ Higgs fields: $\langle \hat{H} \rangle_0 = \varphi^a(\frac{\lambda^a}{2})$ and $\langle \hat{h} \rangle_0 = \tilde{\varphi}^a(\frac{\lambda^a}{2})$, where

$$\begin{aligned} \varphi_2 &= -2\sqrt{m_d m_s}/\lambda_3; \quad \varphi_4 = 2\sqrt{m_d m_s}/\lambda_e; \quad \varphi_6 = 2m_s/\lambda_3; \\ \varphi_3 &= -2m_s/\lambda_3; \quad \varphi_8 = -\frac{2}{\sqrt{3}}(m_b - \frac{m_s}{2})/\lambda_3. \end{aligned} \quad (104)$$

and similarly for the nonvanishing VEV's of the h -Higgs fields:

$$\begin{aligned} \tilde{\varphi}_1 &= \tilde{\varphi}_4 = 2\sqrt{m_u m_c}/\lambda_5; \quad \tilde{\varphi}_6 = -2m_c/\lambda_5; \\ \tilde{\varphi}_3 &= -m_c/\lambda_5; \quad \tilde{\varphi}_8 = -\frac{2}{\sqrt{3}}(m_t - \frac{m_c}{2})/\lambda_5. \end{aligned} \quad (105)$$

Now we have the possibility to calculate the contributions of the horizontal interactions to the amplitudes of following rare processes.

a) pure quark processes: $K^0 \leftrightarrow \bar{K}^0$, $B_d^0 \leftrightarrow \bar{B}_d^0$, $B_s^0 \leftrightarrow \bar{B}_s^0$ oscillations and CP-violating effects in K-, B-, D-meson decays;

b) purely lepton processes: $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\tau \rightarrow \mu\gamma$, etc.

c) quark-lepton processes: $K^\pm \rightarrow \pi^\pm \mu^\pm e^\pm$, $(\mu - e)$ -conversion on nuclei, $K^0 \rightarrow \pi^0 \mu e$, $K^\pm \rightarrow \pi^\pm \nu_i \nu_j$, etc.;

After the calculations the expressions for the $(K_L^0 - K_S^0)$ - and $(D_L^0 - D_S^0)$ - meson mass differences (processes a))take the following general forms:

$$\begin{aligned}\left[\frac{(M_{12})_{12}^K}{m_K}\right]_H &= \frac{1}{2} \frac{g_H^4}{M_{H_0}^4} \left\{ \left[\tilde{\varphi}_a(D \frac{\lambda^a}{2} D^+)_{12} \right]^2 + \left[\varphi_a(D \frac{\lambda^a}{2} D^+)_{12} \right]^2 \right\} f_K^2 R_K, \\ \left[\frac{(M_{12})_{12}^D}{m_D}\right]_H &= \frac{1}{2} \frac{g_H^4}{M_{H_0}^4} \left\{ \left[\tilde{\varphi}_a(U \frac{\lambda^a}{2} U^+)_{12} \right]^2 + \left[\varphi_a(U \frac{\lambda^a}{2} U^+)_{12} \right]^2 \right\} f_D^2 R_D.\end{aligned}\quad (106)$$

Putting into formula (106) the expressions for φ , $\tilde{\varphi}$ (really the second term in the $(M_{12})_{12}^K$ and the first in the $(M_{12})_{12}^D$ are equal to zero) and the elements d_{ij} of the D -mixing matrix, we can obtain the lower limit for the value M_{H_0} . So, we analyze the ratios:

$$\left[\frac{\Delta m_K}{m_K}\right]_H = \frac{g_H^2}{M_{H_0}^2} \text{Re}[C_K] f_K^2 R_K < 7 \cdot 10^{-15} \quad (107)$$

and

$$\left[\frac{\text{Im} M_{12}}{m_K}\right]_H = \frac{1}{2} \frac{g_H^2}{M_{H_0}^2} \text{Im}[C_K] f_K^2 R_K < 2 \cdot 10^{-17}. \quad (108)$$

In formulas (107) and (108) the expression for C_K is as follows :

$$\begin{aligned}C_K = & - \frac{g_H^2}{2\lambda_5^2} \frac{m_t^2}{M_{H_0}^2} \left[\frac{m_c}{m_t} \left(\sqrt{\frac{m_u}{m_c}} + \sqrt{\frac{m_d}{m_s} \frac{m_s}{m_b}} \right) \right. \\ & \left. + i \left(\sqrt{\frac{m_u}{m_c} \frac{m_c}{m_t}} + 3\sqrt{\frac{m_d}{m_s} \frac{m_c}{m_t} \frac{m_s}{m_b}} + 2\sqrt{\frac{m_d}{m_s} \frac{m_s^2}{m_b^2}} \right) \right]^2.\end{aligned}\quad (109)$$

For getting the lower bounds for M_{H_0} from formulas (107) and (108) we can take for the value of $R_K = 1/6 + 1/3(m_K^2/(m_s - m_d)^2)$, $f_K = 0.163 \text{ GeV}$, $m_t = 150 \text{ GeV}$ and $g_H \simeq 0.488$ [9].

In quite an analogous way, we should also write the expression for $M_{12}(B_d)_H$ ($M_{12}(B_d)_H$):

$$\left[\frac{M_{12}(B_d)}{m_{B_d}}\right]_H = \frac{1}{2} \frac{g_H^2}{M_{H_0}^2} C_{B_d} f_{B_d}^2 R_{B_d}. \quad (110)$$

The unitary suppression coefficient will take the following form:

$$C_{B_d} \approx \frac{g_H^2}{2\lambda_5^2} \frac{m_t^2}{M_{H_0}^2} \left[e^{-i\frac{3\pi}{4}} \sqrt{\frac{m_u}{m_c} \frac{m_c}{m_t}} - e^{-i\frac{5\pi}{4}} \sqrt{\frac{m_d}{m_s} \frac{m_c}{m_t}} + \sqrt{\frac{2m_d}{m_s} \frac{m_s}{m_b}} \right]^2.$$

From these formulas and assuming that $(\Delta m(B_d)/m(B_d))|_H \leq (0.73 \pm 0.14)10^{-13}$, $R_{B_d} \simeq 1/6 + 1/3(m_{B_d}^2/(m_b - m_d)^2)$, $f_{B_d} \simeq 0.14 \text{ GeV}$, we also can obtained the lower limits on M_{H_0} .

Note that, if we take the value $f_{B_D} = 0.2 \text{ GeV}$, the lower bounds on the horizontal symmetry breaking scales following from (110) and (4.1) are approximately equal (see (116)).

In this ansatz for the CP- violation parameter $\frac{1}{2} \times \text{Im}(\frac{p}{q})_H$ we have a well-defined magnitude:

$$\frac{1}{2} \times \text{Im}(\frac{p}{q})_{B_d} \approx \frac{\text{Im}M_{12}(B_d)_H}{\text{Re}M_{12}(B_d)_H} \approx 0.3. \quad (111)$$

In SM with this quark mass ansatz the asymmetries will be equal: $A(J/\Psi) \approx -0.34$ and $A(\pi^+\pi^-) \approx -0.44$, respectively. The contributions of CP-violating horizontal interactions to the asymmetries for both B^0 -decays are identical but the signs differ (in this approach $\max|A_f| \approx 0.17$) .

Finally, let us give the useful estimate:

$$\left[\frac{\Delta m_{B_s}}{\Delta m_{B_d}} \right]_H \approx \left[\frac{V_{ts}}{V_{td}} \right]^2 \approx \frac{1}{2} \frac{m_s}{m_d} \times (1 + \frac{m_c m_b}{m_t m_s})^2 \approx 17 \div 20. \quad (112)$$

As follows from the expression (106) in the approach with the ansatz (101) the value of the $(D_L^0 - D_S^0)$ - mass difference will be considerably suppressed by the unitarity coefficient $(U\lambda^8 U^+)_{12}$ (see (102) comparing with the similar coefficient $(D\lambda^8 D^+)_{12}$ in (103)). But it is very important to note that in our approach with the symmetric ansatz [8] this suppression for the Δm_D - mass difference will be absent. From formulas (106) it's possible to get in this ansatz the next approximate expression for the $D_L^0 - D_S^0$ and $K_L^0 - K_S^0$ - mass difference ratio:

$$\frac{\Delta m_D}{\Delta m_K} \approx \frac{m_D}{m_K} \frac{f_D^2 R_D}{f_K^2 R_K} \frac{[\tilde{\varphi}_8^{sym}]^2}{[\varphi_8^{sym}]^2}. \quad (113)$$

From this expression it follows that for the corresponding values of the VEV's ratio $-\tilde{\varphi}_8/\varphi_8$ - the magnitude for $D_L^0 - D_S^0$ - mass difference could be considerable so that

$$\rho_D \longrightarrow \rho_{exp} = 5 \cdot 10^{-3}. \quad (114)$$

This is a characteristic feature of the horizontal model with symmetric ansatz in which we could get the considerable magnitude for $\bar{D}^0 - D^0$ - mixing and this is very intriguing for the future experiments.

After all, in this section we calculate the branching ratio for the $\mu \rightarrow 3e$ -decay (process b)). For this we have proposed the form of the charged lepton mass matrix - the one that was used for down- quarks. Using this ansatz for a charged lepton mixing, we find out that:

$$\begin{aligned} Br(\mu \rightarrow 3e) &\simeq 12 \frac{g_H^4}{g_W^4} \frac{M_W^4}{M_{H_0}^4} \left\{ f^{abc} \left(L \frac{\lambda^a}{2} L^+ \right)_{21} (g_H \phi^b) f^{a'b'c} \left(L \frac{\lambda^{a'}}{2} L^+ \right)_{11} (g_H \phi^{b'}) \right\}^2 \\ &\simeq \frac{3}{2} \frac{g_H^4}{g_W^4} \frac{M_W^4}{M_{H_0}^4} \frac{g_H^4}{\lambda_5^4} \frac{m_t^4}{M_{H_0}^4} \left[-\frac{1}{\sqrt{2}} \frac{m_c^2}{m_t^2} \sqrt{\frac{m_u}{m_c}} + \frac{m_c}{m_t} \left(\sqrt{\frac{m_u}{m_c}} - \sqrt{\frac{2m_e}{m_\mu}} \right) \frac{m_\mu}{m_\tau} \right]^2 \end{aligned} \quad (115)$$

where $(\phi = \tilde{\varphi}^a, \varphi^a)$ are the nonvanishing VEV's of the h^a - and H^a -Higgs fields, respectively.

Using the experimental information for the rare processes a) and b) and the formulas (107), (108), (110), (116) and (115) depending on the Yukawa coupling we can get very small values for breaking scale, M_{H_0} , of the $SU(3_H)$ gauge symmetry:

$$\begin{aligned} M_{H_0} &> 0.2TeV - 0.5TeV, (\lambda_5 \simeq O(1)), \\ M_{H_0} &> 0.6TeV - 1.5TeV (\lambda_5 \simeq O(0.1)). \end{aligned} \quad (116)$$

4.2 The SUSY $SU(3)_H$ -gauge model with "Democratic" ansatz for quark and lepton families

In the electroweak $SU(2) \times U(1)$ - model it is impossible to define separately the "mixings" in up- and down-quark mass matrices ("absolute mixing"). The SM still provides a certain freedom in choosing the primary mass matrices for quarks in such a way as to get large mixings both for up-, U , and down-quarks D , provided $UD^+ = V_{CKM}$. To get information on this "absolute mixing", one should investigate rare processes in the framework of the horizontal gauge model. Therefore, it would be very interesting to consider a scheme, in which these mixings are large, and to study possible constraints on the horizontal symmetry breaking scale M_{H_0} . So, we consider the "up"- and "down"- fermion mass matrices of the following ("democratic") form, which has been used for explaining the special role of the t - quark mass ($m_t \sim \Lambda_{EW} \gg m_c, m_u$) [40, 41]:

$$\mathbf{M}_f^0 = \frac{1}{3}m_f \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (117)$$

A BCS theory of quark generation could explain such form of mass matrix. The term BCS mechanism is used to refer to Cooper pair formation through attractive forces between some constituent ur-fermions [40, 41].

To obtain the democratic form of quark mass matrices in the model with the family gauge symmetry, it is necessary to consider, besides H - and h - superfields (see (72)), the additional superfields $H_0(1_H, 2_L, -\frac{1}{2})$ and $h_0(1_H, 2_L, \frac{1}{2})$ and to conserve the $S(3)_L \times S(3)_R$ vacuum symmetry. The diagonalization of the democratic matrices yields a mass gap, i.e. the masses of t - or b - quarks are split far apart from all other degenerate masses of c -, u - or s -, d -quarks. The complete mass matrices are $M_f = M_f^0 + \Delta M_f$. As a result of the diagonalization, they yield the physical fermion mass matrices M^D for "up" and "down" quarks: $M_f^D = V_f^+ M_f V_f$, where $V_f = V_{f0} V_{f1}$ and $V_{di} = D_i$, $V_{ui} = U_i$, $i = (0, 1)$. Here V_{f0} ($f = d \text{ or } u$) are unitary matrices constructed from the eigenvectors of the M_f^0 -matrix and equal to :

$$(\mathbf{U}^0)^T = (\mathbf{D}^0)^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \quad (118)$$

In the first approximation, there is a conservation of the isotopic symmetry of the mixing mechanism in the up" and "down" quark mass matrices. The matrices U_1 and D_1 ($U_1 \neq D_1$) are small corrections to produce the correct form of the V_{CKM} -matrix. Using the U_1 - and D_1 - correction matrices, we can construct the mass matrices M_f differing from M_f^0 by a small correction factor.

Let us construct explicitly the corresponding splitting of the horizontal gauge boson mass matrix. In this case, each of the 8×8 -dimensional mass matrices: $\left[\Delta M_H^2\right]_d^{ab}$ and $\left[\Delta M_H^2\right]_u^{ab}$, $a, b = 1, 2, \dots, 8$, is broken into 3×3 - and 5×5 -dimensional matrices. The additional contributions to the mass spectra of the $H_{2,5,7}^\mu$ - and $H_{1,4,6,3,8}^\mu$ - horizontal gauge bosons take, correspondingly, the following forms:

$$\left[\Delta M_H^2\right]_f^{a,b=2,5,7} = \frac{g_H^2 m_f^2}{12\lambda_f^2} \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}, \quad (119)$$

$$\left[\Delta M_H^2\right]_f^{a,b=1,4,6,3,8} = \frac{g_H^2 m_f^2}{36\lambda_f^2} \begin{pmatrix} 2 & -1 & -1 & 0 & 2\sqrt{3} \\ -1 & 2 & -1 & 3 & -\sqrt{3} \\ -1 & -1 & 2 & -3 & -\sqrt{3} \\ 0 & 3 & -3 & 6 & 0 \\ 2\sqrt{3} & -\sqrt{3} & -\sqrt{3} & 0 & 6 \end{pmatrix}. \quad (120)$$

The diagonalization of these mass matrices can easily be realized by the orthogonal matrices $O^{(-)}$ and $O^{(+)}$: so that $Z_a^{\mu(-)} = O_{ab}^{(-)} H_b^{\mu(-)}$ ($a, b = 2, 5, 7$) and $Z_a^{\mu(+)} = O_{ab}^{(+)} H_b^{\mu(+)}$ ($a, b = 1, 4, 6, 3, 8$). In accordance with expressions (119) and (120), let us write down the forms of the $O_{ab}^{(-)}$ - and $O_{ab}^{(+)}$ - diagonalizing matrices:

$$\mathbf{O}_{ab}^{(-)T} = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \quad (121)$$

$$\mathbf{O}_{ab}^{(+)T} = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{2}{3} & 0 & \frac{\sqrt{2}}{3} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{3} & \frac{1}{\sqrt{6}} & -\frac{1}{3\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{3} & -\frac{1}{\sqrt{6}} & -\frac{1}{3\sqrt{2}} \\ 0 & \frac{1}{\sqrt{3}} & 0 & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & \frac{\sqrt{3}}{3} & 0 & \sqrt{\frac{2}{3}} \end{pmatrix}. \quad (122)$$

Note that the signs $(-)$ and $(+)$ indicate the opposite CP- transformation properties of the $J_{H_{2,5,7}}^\mu$ - and $J_{H_{1,4,6,3,8}}^\mu$ - gauge horizontal currents for each value of the index μ ($\mu = 0 \text{ or } 1, 2, 3$). Until there is no mixing between these currents, there may be no CP- violation in the gauge sector of the horizontal interactions.

As a result of this approach, we get a very simple splitting between 8- gauge Z_a^μ - bosons:

$$\begin{aligned} M_{Z_1}^2 &= M_{Z_4}^2 = M_{Z_6}^2 = M_{Z_7}^2 = M_{H_0}^2 \\ M_{Z_3}^2 &= M_{Z_8}^2 = M_{Z_2}^2 = M_{Z_5}^2 = M_{H_0}^2 + \frac{g_H^2}{4} \sum_f \frac{m_f^2}{\lambda_f^2}. \end{aligned} \quad (123)$$

The mass spectra of $Z_{1,4,6,7}^\mu$ - gauge bosons correspond to the global $SU(2)_H \times U(1)_{H-}$ symmetry in the gauge sector, which was considered in sections 3 and 4.

If we use the family mixing (100), the Lagrangian for the quark- gauge boson interactions will be

$$\begin{aligned} \mathcal{L}_Q &= \frac{g_H}{2} \bar{Q} \gamma_\mu \left(\left[-D_1 \lambda^8 D_1^+ \right] \tilde{Z}_1^\mu \right. \\ &+ \left[\frac{\sqrt{3}}{2} D_1 \lambda^3 D_1^+ - \frac{1}{2} D_1 \lambda^1 D_1^+ \right] \tilde{Z}_4^\mu - \left[\frac{1}{2} D_1 \lambda^3 D_1^+ + \frac{\sqrt{3}}{2} D_1 \lambda^1 D_1^+ \right] \tilde{Z}_6^\mu \\ &+ \left[\frac{1}{2} D_1 \lambda^4 D_1^+ - \frac{\sqrt{3}}{2} D_1 \lambda^6 D_1^+ \right] \tilde{Z}_3^\mu + \left[\frac{1}{2} D_1 \lambda^6 D_1^+ + \frac{\sqrt{3}}{2} D_1 \lambda^4 D_1^+ \right] \tilde{Z}_8^\mu \\ &\left. + \left[D_1 \lambda^5 D_1^+ \right] \tilde{Z}_2^\mu - \left[D_1 \lambda^7 D_1^+ \right] \tilde{Z}_5^\mu - \left[D_1 \lambda^2 D_1^+ \right] \tilde{Z}_7^\mu \right) Q, \end{aligned} \quad (124)$$

where one has $Q = Q_d = (d, s, b)$, or $Q = Q_u = (u, c, t)$.

At this expression we take a certain small quark mixing (the "democracy" is broken) but we will not consider the additional gauge boson mixing and just assume that $M_{Z_a} \approx M_{\tilde{Z}_a}$, $a=1,2,3..8$. For our purpose, it will suffice to take into account only a new small correction to the quark family mixing $-D_1$ and U_1 matrices, leading to the correct form of the CKM- matrix for charged EW currents. We may consider the chain of symmetry breaking from the original $U(3)_L \times U(3)_R$ of the massless quarks and leptons:

$$U(3)_L \times U(3)_R \xrightarrow{m_{t,b} \neq 0} S(3)_L \times S(3)_R \xrightarrow{m_{c,s} \neq 0} S(2)_L \times S(2)_R \xrightarrow{m_{u,d} \neq 0} 1 \quad (125)$$

At the first stage only the third family is massive and the other two are massless. At the second stage only one generation remains massless. At last, at the third stage the first generation also gets mass. For instance, we may consider the up- and down- quark mass matrices like (100) and (101):

$$\mathbf{M}_u^{\text{dem}} \longrightarrow \frac{1}{3} m_t \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_c}{6} u_1 \begin{pmatrix} 1 & 1 & u_2 \\ 1 & 1 & u_2 \\ u_2 & u_2 & -2 \end{pmatrix} + \sqrt{\frac{m_u m_c}{3}} u_3 \begin{pmatrix} 1 & 0 & u_4 \\ 0 & -1 & -u_4 \\ u_4 & -u_4 & 0 \end{pmatrix} \quad (126)$$

and

$$\mathbf{M}_d^{\text{dem}} \longrightarrow \frac{1}{3}m_b \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_s}{6}d_1 \begin{pmatrix} 1 & 1 & d_2 \\ 1 & 1 & d_2 \\ d_2 & d_2 & -2 \end{pmatrix} + \sqrt{\frac{m_d m_s}{3}}d_3 \begin{pmatrix} 1 & -\frac{i}{\sqrt{2}} & d_4 \\ \frac{i}{\sqrt{2}} & -1 & -d_4 \\ d_4^* & -d_4^* & 0 \end{pmatrix} \quad (127)$$

where

$$\begin{aligned} u_1 &= 2\sqrt{2} - 1, \quad u_2 = -\frac{4 + \sqrt{2}}{2\sqrt{2} - 1}, \quad u_3 = \sqrt{2} - 1, \quad u_4 = \frac{\sqrt{2} + 1}{2 - \sqrt{2}}; \\ d_1 &= 1 + 2\sqrt{2}, \quad d_2 = \frac{4 - \sqrt{2}}{1 + 2\sqrt{2}}, \quad d_3 = \sqrt{2}, \quad u_4 = \frac{1 + i\sqrt{2}}{2}. \end{aligned} \quad (128)$$

Now, for the further estimates of the $SU(3)_H$ - symmetry breaking scale M_{H_0} we use the results from section 4 and the following useful relation

$$\sum_a (DT^a D^+)_{ik} (DT^a D^+)_{mn} = \frac{1}{2} \left(\delta_{in} \delta_{km} - \frac{1}{3} \delta_{ik} \delta_{mn} \right). \quad (129)$$

Then, the expression for the K_L^0 - K_S^0 meson mass difference (90), derived from formulas (125), (122) and (124), will change only due to the new suppression coefficient $\sim m_f^2/M_{H_0}^2$, e.g.

$$\left\{ \frac{\Delta m_K}{m_K} \right\}_H = \frac{g_H^2}{M_{H_0}^2} \left\{ \frac{g_H^2}{4M_{H_0}^2} \sum_f \frac{m_f^2}{\lambda_f^2} \right\} \text{Re}(C_K^0) f_K^2 R_K. \quad (130)$$

where $C_K^0 = \sum_a' (D_1 T^a D_1^+)_{21} (D_1 T^a D_1^+)_{21}$, and the index (ι) indicates that summation is only over the diagrams with the exchanges of $Z_1^\mu, Z_4^\mu, Z_6^\mu, Z_7^\mu$ - gauge horizontal bosons. In this approximation, the lower bound on the local horizontal symmetry breaking scale may be smaller than in case (i) from subsection 3.4 (the $SU(2)_H \times U(1)_H$ symmetry). For instance, if we assume that $m_f = m_t$ and make our usual assumption for the relation g_H/λ_f , we can get:

$$M_{H_0} \approx \sqrt{\frac{g_H}{2\lambda_f}} \sqrt{\frac{m_t}{M_H}} \times M_H > O(0.8) TeV. \quad (131)$$

In the last inequality we use the estimate for $M_H > 8 - 9 TeV$ taken from Table 12. Note, that the consequences for the $B_{d,s}^0 \longleftrightarrow \bar{B}_{d,s}^0$ oscillations remain as in Table 12- e.g., this value for the gauge symmetry breaking scale corresponds to the present quantity of the $B_{d_1}^0$ - $B_{d_2}^0$ - meson mass difference.

Really, one could expect a very low horizontal local symmetry breaking scale- M_{H_0} in pure quark (or pure lepton) rare processes due to the changes of the quantum numbers of generations therein: $|\Delta H| \neq 0$. From the experimental limits on the amplitudes of quark-

lepton rare processes, where $\Delta H = 0$, we obtain considerably larger values for this scale (95),(97). What are the consequences of the studies of the lower bound on the horizontal local symmetry breaking scale ? Here are some of them:

1. The most pronounced processes promoting the discovery of a new hypothetic interaction are quark- lepton rare processes like $K \longrightarrow \pi + \mu + e^-$, or the μ/e^- conversion on nuclei. Within this class, the decay $K \longrightarrow \pi + \nu_i + \nu_j$ may also turn out to be very important. In this case, the "traditional" construction for the quark- lepton families has been assumed:

$$C_1 = (Q_1[u, d]; L_1[\nu_e, e]), C_2 = (Q_2[c, s]; L_2[\nu_\mu, \mu]), C_3 = (Q_3[t, b]; L_3[\nu_\tau, \tau])$$

and here the mixings between Q_i - and L_j - families are not very large. Then we should think that the local horizontal symmetry breaking scale is very large, as follows from the limits (95) - e.g., it may be more than 60 TeV. For this large enough scale, the contributions to the pure quark rare reactions (meson mixings), or pure lepton rare decays ($\mu \rightarrow 3e$ etc.), from these forces will be very small. In particular, if the splitting between the masses of 8- gauge horizontal bosons is as in our previous examples: $|(\Delta M_H^2)_a| \ll M_H^2$. In the case of the large splitting $|(\Delta M_H^2)_a| \gg \min M_{H^a}^2$, we may use, for practical purposes, the results of the $U(1)_{T_3} \times U(1)_{T_8}$ -gauge group, where it was established that the lower limits on the M_H -scale are: $M_H > 170 - 195 \text{ TeV}$ (Δm_K , Table 12); $M_H > 60 - 100 \text{ TeV}$ (from the μ/e^- conversion on nuclei); and $M_H > 35 \text{ TeV}$ (from $K \longrightarrow \pi + \mu + e$ if $B \leq 10^{-10}$ (97)), or $M_H > 100 \text{ TeV}$ (if the limit 10^{-12} is reached in the nearest future in BNL- experiment). The lower bound on M_H obtained from the modern experimental limit on a pure lepton rare decay, like $\mu \rightarrow 3e$, is compatible with the bounds resulting from the $K \longrightarrow \pi + \mu + e^-$ experiment. So, we have $M_H > 28 \text{ TeV}$ (93).

2. An alternative scenario we have to consider is connected with searching for another possible mechanism of the (q-l) - mixing to diminish the scale M_H to the values approaching the region of (1-10) TeV. For this purpose, we may also use an indefinite correlation both between the Q_i - and L_j - family mixings, and, within L_j - lepton families, - between charged lepton and neutrino mixings, so far as the experimental situation allows us to do so. These explicit differences in the origin of quark and lepton mass spectra make one also suppose that leptonic families might mix by quite a different mechanism, different from the above example of quark mixing. One should also remember that in the SM it is impossible, in principle, to establish a correlation between the Q_i -quark and L_j - lepton mixings. Due to electroweak interactions, we can only get information on the correlations between up- and down- quark mixing. But now there is still a certain freedom in the choice of the mixing models for charged leptons or neutrino, especially in the case of very small neutrino masses.

From the analysis of pure quark (lepton) rare processes in the gauge horizontal model we may get complete information about separate "absolute" mixings of up- quarks (neutrinos) and down- quarks (charged leptons). And from quark- lepton rare reactions in the frames of gauge horizontal interactions we may define correlations between $[d, s, b]$ ($[u, c, t]$) quark- and $[e, \mu, \tau]$ or $[\nu_e, \nu_\mu, \nu_\tau]$ - lepton bases. In the above examples (see section 4), the supposition of the absence of correlation between down- quark and charged lepton mixings resulted in rather high limits for the M_H -scale, obtained from quark- lepton pro-

cesses, compared to those from pure "q", or pure "l" -processes. Now let us consider the scheme when the magnitude of correlation between quark- and charged lepton mixings is large.

$$\begin{aligned}
\mathcal{L}_l = & \frac{g_H}{2} \bar{\Psi}_l \gamma_\mu \left(\left[\frac{\sqrt{3}}{2} L_1 \lambda^3 L_1^+ + \frac{1}{2} L_1 \lambda^8 L_1^+ \right] Z_1^\mu \right. \\
& + \left[\frac{\sqrt{3}}{4} L_1 \lambda^3 L_1^+ - \frac{3}{4} L_1 \lambda^8 L_1^+ - \frac{1}{2} L_1 \lambda^6 L_1^+ \right] Z_4^\mu \\
& - \left[\frac{1}{4} L_1 \lambda^3 L_1^+ - \frac{\sqrt{3}}{4} L_1 \lambda^8 L_1^+ + \frac{\sqrt{3}}{2} L_1 \lambda^6 L_1^+ \right] Z_6^\mu \\
& + \left[-\frac{\sqrt{3}}{2} L_1 \lambda^1 L_1^+ + \frac{1}{2} L_1 \lambda^4 L_1^+ \right] Z_3^\mu + \left[\frac{1}{2} L_1 \lambda^1 L_1^+ + \frac{\sqrt{3}}{2} L_1 \lambda^4 L_1^+ \right] Z_8^\mu \\
& \left. - \left[L_1 \lambda^5 L_1^+ \right] Z_2^\mu + \left[L_1 \lambda^2 L_1^+ \right] Z_5^\mu - \left[L_1 \lambda^7 L_1^+ \right] Z_7^\mu \right) \Psi_l, \tag{132}
\end{aligned}$$

where $\Psi_l = (e, \mu, \tau)$.

It is obvious that the lower bound on M_{H_0} can also be very small ($\sim O(1)TeV$) as far as pure charge lepton rare processes are considered (e.g., the modern high experimental limit for the $\mu^+ \rightarrow e^+ + e^+ + e^-$ - decay). Again, in this approach there appears a similar additional suppression factor $\sim m_f^2/M_H^2$ for the partial width of this process. So, assuming that $m_f \approx m_t$ and $(L_1 T^3 L_1^T)_{21} (L_1 T^3 L_1^T)_{11} = L^3 \approx \sqrt{m_e/m_\mu}$, as was accepted in formula (93) (Fritzsch ansatz for lepton mixing), we have: $M_{H_0} > \sqrt{g_H/(2\lambda_f)} \sqrt{m_f M_H} > O(1)TeV$ for $M_H > 7.5TeV$.

Besides, in this model we could obtain lower limits for the horizontal symmetry breaking scale by analyzing quark- lepton rare processes like $K \rightarrow \pi + \mu + e^-$, or the μ/e -conversion. For example, for process of the first type the estimate (97) is:

$$\begin{aligned}
M_{H_0} & \approx \sqrt{2|d_{13}|} M_H (K^+ \rightarrow \pi^+ + \mu^+ + e^-), \\
M_{H_0} & \approx \sqrt{2|d_{12}d_{23}|} M_H (K^+ \rightarrow \pi^+ + \mu^- + e^+). \tag{133}
\end{aligned}$$

If we take the values for $(D_1)_{ij} = d_{12}, d_{23}, d_{13}$ from all the three ansatzes, we may verify (97) that the scale M_{H_0} will be rather low: $M_{H_0} > O(1TeV)$ for $M_H > 35TeV$. The last estimate is conditioned by the model-dependent value of d_{13} , the latter being not very precisely defined from the comparison with the V_{cb} - matrix element. Here we assume that $|d_{13}| \ll |V_{cb}|$, which does not contradict the experiment. Note, in this scheme we make a very interesting prediction for the heavy quark-, or heavy τ - lepton rare decay. For example, for this horizontal symmetry breaking scale ($M_{H_0} \sim 1TeV$) the partial width for the τ - lepton decay - $\tau \rightarrow \mu + d + \bar{s}$ may be rather large $\sim 10^{-5}$.

By analogy, we find that the rate for μ to e - conversion (94) is reduced by the factor $2|d_{12}d_{13}|$. Combining this factor and the expression (94), we come to the following limit for M_{H_0} :

$$M_{H_0} \approx \sqrt{2|d_{12}d_{13}|} M_H > O(1-2) TeV. \quad (134)$$

In this scheme the bounds (133) and (134) differ by the factor of $\sqrt{|d_{12}|}$. Note, that our earlier limits for M_H (97) from for these two very important processes also differ by the same factor $\approx 2-2.5$.

Finally, we have to consider the decay $K \longrightarrow \pi + \nu + \nu$. Now the experimental lower limit for the partial width of this process is :

$$Br(K \longrightarrow \pi + \nu_i + \nu_j) < 5 \times 10^{-9}.BNL. \quad (135)$$

According to (135), the immediate estimate of this decay in our approach gives us the following constraint: $M_{H_0} > 10TeV$. To lower this limit, it's necessary to elucidate the origin of the neutrino mass spectrum. Clearly, to achieve this one has to consider an extension of the fermion matter spectrum of new particles- first of all, new neutral neutrino- like particles ($T_{SU(2)} = \frac{1}{2}, SU(3)_H$ -singlet). This could result in an efficient decrease in the value of the coupling constant g_H in the neutrino horizontal interaction. The studies of the regularities in the observed mass spectrum of ordinary particles might indicate possible existence of new particles like those occurring in GUT's (E(6)) or in other earlier extensions of the SM (Left- Right models with mirror particles or with the fermion spectrum doubled).

5 Discussion. The quark-lepton nonuniversal character of the local family interactions

The main point of our considerations in this paper to study the next chain:

The nature of quark and lepton masses \implies
The quark - lepton family mixing \implies
A new family dynamics at 1 TeV energy.

In chapter 2 we have considered the rank eight GUST with gauge symmetry $G = SU(5) \times U(1) \times (SU(3) \times U(1))_H \subset SO(16)$ ($G = SO(10) \times (SU(3) \times U(1))_H \subset SO(16)$). GUSTs originating from level-one KMA contain only low-dimensional representations of the unification group. It is, therefore, difficult to break the gauge symmetry. In order to solve this problem we have considered as the observable gauge symmetry the diagonal subgroup G^{sym} of rank 16 group $G \times G \subset SO(16) \times SO(16) (\subset E(8) \times E(8))$. This

construction allows us to break the GUST symmetry to the low energy gauge group, which includes the G_H family gauge symmetry. The virtue of the $GUST^{symm}$ considered is that its low-energy spectrum does not contain particles with exotic charges. This GUST which is based on the maximal invariant $SO(16)$ subgroup of $E(8)$ leads to the 3_H light and 1_H heavy families, and the GUST construction which is based on the $E(6) \times SU(3)_H \subset E(8)$ ($SU(3)^4 \subset E(6)$) gauge symmetry and which predicts only three light quark-lepton families.

For example the model with basis vectors b_1, b_2, b_4 of Model 1 can contain the $E(6) \times SU(3)$ -sublattice, which corresponds to the simple roots:

$$\varepsilon_i - \varepsilon_{i+1}, \quad i = 1 - 4, 6 - 8; \quad \varepsilon_4 + \varepsilon_5; \quad -1/2 \sum_{i=1}^8 \varepsilon_i, \quad i = 1 - 8$$

in the orthogonal basis ε_i , $i = 1 - 8$. Where states from RNS-sector correspond to the first seven roots and state from b_2 -sector (Mod.1) $\psi_{1/2}^\mu |0\rangle_L \otimes \prod_{i=1}^8 \Psi_i^* |0\rangle_R$ corresponds to the last root. But we must select the $SU(3)$ -factor from this model. We can add the b_2 -vector of the Model 2 or 3 for this. However in this case we can not fix the $E(6) \times SU(3)$ -lattice since the lattice destroys or grows. And this way does not lead to three generations. In this point it's seems more perspective to work with real world-sheet fermions and with less rank groups.

A variant for unusual nonuniversal family gauge interactions of known quarks and leptons could be realized if we introduce into each generation new heavy quarks ($F = U, D$), and leptons (L, N) singlets (it is possible to consider doublets also) under $SU(2)_L$ - and triplets under $SU(3)_H$ -groups. (This fermion matter could exist in string spectra. See the all three models with $SU(3_H) \times SU(3_H)$ family gauge symmetry). Let us consider for concreteness a case for charged leptons: $\Psi_l = (e, \mu, \tau)$ and $\Psi_L = (E, M, T)$. Primarily, for simplicity we suggest that the ordinary fermions do not take part in $SU(3)_H$ -interactions ("white" color states). Then the interaction is described by the relevant part of the SUSY $SU(3)_H$ -Lagrangian and gets the form

$$\mathcal{L}_H = g_H \bar{\Psi}_L \gamma_\mu \frac{\Lambda^{a_{6 \times 6}}}{2} \Psi_L O_{ab} Z_\mu^b, \quad (136)$$

where

$$\Lambda^{a_{6 \times 6}} = \begin{pmatrix} S(L\lambda^a L^+)S & -S(L\lambda^a L^+)C \\ -C(L\lambda^a L^+)S & C(L\lambda^a L^+)C \end{pmatrix}.$$

Here we have $\Psi_L = (\Psi_l; \Psi_L)$. The matrix O_{ab} ($a, b = 1, 2, 3, \dots, 8$) determines the relationship between the bare, H_μ^b , and physical, Z_μ^b , gauge fields. The diagonal 3×3 matrices $S = \text{diag}(s_e, s_\mu, s_\tau)$ and $C = \text{diag}(c_e, c_\mu, c_\tau)$ define the nonuniversal character for lepton horizontal interactions, as the elements s_i depend on the lepton masses, like $s_i \sim \sqrt{m_i}/M_0$ ($i = e, \mu, \tau$). The same suggestion we might accept for local quark family interactions.

For the family mixing we might suggest the next scheme. The primary 3x3 mass matrix for the light ordinary fermions is equal to zero : $M_{ff}^0 \approx 0$. The 3x3- mass matrix for heavy fermions is approximately proportional to unite matrix: $M_{FF}^0 \approx M_0^Y \times 1$, where $M_0^Y \approx 0.5 - 1.0 TeV$ and might be different for F_{up-} , F_{down-} quarks and for F_L - leptons. We assume that the splitting between new heavy fermions in each class F_Y (Y=up, down, L) is small and, at least in quark sector, might be described by the t- quark mass. Such we think that at first approximation it is possible to neglect by the heavy fermion mixing. The mixing in the light sector is completely explained by the coupling light fermions with the heavy fermions. As a result in of this coupling the 3x3- mass matrix M_{fF}^0 could be constructed by "democratic" way which could lead to the well known mass family hierarchy:

$$M_{6 \times 6}^0 = \begin{pmatrix} M_{ff}^0 & M_{fF}^0 \\ M_{Ff}^0 & M_{FF}^0 \end{pmatrix},$$

where

$$M_{fF}^0 \approx M_{fF}^{dem} + M_{fF}^{corr}. \quad (137)$$

The diagonalization of the M_{fF}^0 - mass matrix $XM_{fF}^0X^+$ (X = L-, D-, U- mixing matrices) gives us the eigen values, which are to define the family mass hierarchy- $n_1^Y \ll n_2^Y \ll n_3^Y$ and the following relations between the masses of the known light fermions and a new heavy mass scale:

$$n_i^Y = \sqrt{m_i M_0^Y}, \quad i = 1_g, 2_g, 3_g; \quad Y = up-, down - fermions. \quad (138)$$

In this "see-saw" mechanism the common mass scale of new heavy fermions might be not very far from the $\sim 1 TeV$ energy, and as a consequence of the last the mixing angles s_i - might be not too very small. There is another interesting relation between the mass scales n_i^Y might be in this mechanism, at least for the quark case:

$$\begin{aligned} n_t/n_c &= n_c/n_u = q_H^u, & q_H^u &\approx 14 - 16, \\ n_b/n_s &= n_s/n_d = q_H^d, & q_H^d &\approx 4 - 5. \end{aligned}$$

As an explicit example of non-universal $SU(3_H) \times SU(3_H)$ local family interactions could be considered the model 3 (see section 2).

In this approach we get the suppression for quark-lepton flavour changing processes , like μ to e - conversion, $K \rightarrow \pi + \mu + e$ - or $K \rightarrow \pi + \nu + \nu$ - decays. And as result we can hope to get the very low bound for the horizontal gauge boson masses (in some TeV range).

Acknowledgements

One of us (G.V) would like to thank INFN for financial support and the staff of the Physics Department in Padova, especially, Professor C. Voci for warm hospitality during his stay in

Padova. Also, he is pleased to the University of Padova and to the Physical Department in Trieste and to Professor G. Costa and Professor N. Paver for hospitality and the financial support. It is a great pleasure for him to thank Professor G. Costa, Professor L. Fellin, Professor J.Gasser, Professor G. Harrigel, Professor H. Leutwyller, Professor P.Minkowski, Professor D. Nanopoulos, Professor N. Paver, Professor V. Petrov, Professor P. Ramond, Professor P. Sorba, Professor M. Tonin and Professor C. Voci for useful discussions and for the help and support. Many interesting discussions with colleagues from Theory Department in Padova INFN Sezione, especially, to Professor S. Sartory, during this work are also acknowledged. Finally, he would like to express his gratitude to Professor P. Drigo and their colleagues of the Medical School of the University of Padova.

The research described in this publication was made possible in part by Grants No RMP000 and RMP300 from the International Science Foundation.

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